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Minimum Effort Pursuit Guidance with Delayed Engagement Decision

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The paper proposes a guidance algorithm for a single pursuer facing multiple maneuvering evaders, one of which is finally engaged by the pursuer. It is assumed that the pursuer makes a decision to engage one of the evaders at a given moment in flight, and the probability of each possible choice is known in advance. Under these assumptions, a guidance law is derived that optimizes the expected integral quadratic control effort, where the expectation is taken over all possible engagement decisions. The derived guidance law turns out to be a linear combination of optimal pursuit guidance laws toward each of the evaders separately, with time-dependent coefficients. Numerical simulation results show the efficiency of the proposed guidance law with respect to conventional solutions that do not take into account the delayed engagement decision.

I. Introduction

AN INCREASING interest in the guidance design literature has concentrated lately on issues involving several targets (evaders) and several interceptors (pursuers). Many recent publications assumed a form of cooperation between the interceptors and attempted to use cooperative action to improve the effectiveness of the interceptors. This can be realized by achieving a favorable impact angle geometry between the attackers and the target [1] or simultaneous impact time as in [2,3]. Similar approaches were adopted in contributions such as [4,5], to cite only a few that tackled this problem.

Although dealing with multiple target scenarios is of a clear practical interest, the guidance design literature on this subject is much less extensive. A considerable part of the contributions tackling this type of scenarios is geared toward target assignment problems such as [6,7]. Few contributions consider interception guidance algorithms in the presence of multiple targets. For example, [8] proposes such an algorithm assuming that the interceptor is supposed to "intercept" the targets successively at given times. Another contribution [9] examines the performance of the classical proportional navigation guidance algorithm in the presence of multiple targets.

In this paper, the problem is formulated and solved using a linearized model of the engagement. In this framework, the solution turns out to be a weighted sum of optimal pursuit strategies toward each of the evaders with weighting coefficients that are dependent on time-to-go values to each target and on the probabilities for the engagement decision. For constant-acceleration evaders these strategies are augmented proportional navigation (APN) guidance commands. This outcome can readily be implemented, and we examine the optimal strategy performance by numerical simulation in linearized, but especially in nonlinear, engagement models.

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II. Problem Statement and Modeling

The scenario considered in this paper is illustrated in Fig. 1. It consists of a pursuer that faces with a number of evaders (two in the illustrating figure) at the first stage of the engagement, and then chooses a single evader to pursue.

It is assumed that the pursuer is part of a salvo of missiles that have been launched against the same evaders and that its teammates may have destroyed some of the evaders before the pursuer has to decide which of the evaders to engage. It is also assumed that at a given time moment, called decision time and denoted t_d, the pursuer will select one of the evaders, in principle the one of highest priority among those that are still alive. Consider, for example, the case of two evaders. Assume that E_1 is already under pursuit by a pursuer P_0 launched before and that the pursuit of E_1 is planned to be completed at time t_d. Assume also that E_1 is a higher priority evader than E_2. In the case that E_1 is not destroyed by P_0, the pursuer P will have to engage E_1 because it is a higher priority evader so that the pursuit of E_2 is left to a trailing pursuer. The chance that P will have to go after E_1 is equal to the probability p that P_0 will fail to destroy E_1. Then there is indeed a probability p that P will go after $E1$ and a probability $1 - p$ that P will go after E_2. In general, assuming that the probability of destroying each of the evaders is known and that the priority order of the evaders is known, it is possible to determine the probability for any given evader to be engaged by the pursuer.

The problem addressed in this paper is how to design the guidance law of the pursuer before the assignment decision in order to minimize the expected guidance effort over all the possible outcomes of the engagement decision.

A. Engagement Model and Its Linearization

A planar engagement between the pursuer and N evaders is considered. It is assumed that all adversaries have constant speeds.

In Fig. 2, the schematic engagement geometry is depicted in a fixed coordinate system. The origin of the coordinate system is collocated with the pursuer's initial position, and the x axis is in the direction of the initial line of sight to the first evader. We assume that the initial lines of sight from the pursuer to all evaders more or less coincide. The points P and E_j denote the current positions of the pursuer and the evaders, respectively. Their coordinates are (x_p, y_p) and (x_{e_j}, y_{e_j}), $j = 1, \ldots, N$; a_p, a_{e_j} are their lateral accelerations normal to the velocity vectors V_p, V_{e_j}. Let φ_p denote the angle between the velocity vector V_p of the pursuer and the x axis; $\varphi_{e_j}, j = 1, \ldots, N$, be the angles between the velocity vectors V_{e_j} of the evaders and the x

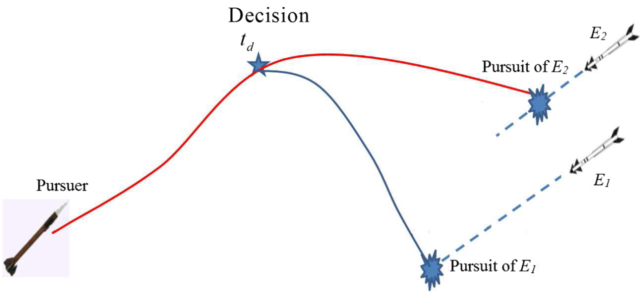

Fig. 1 Pursuit scenario.

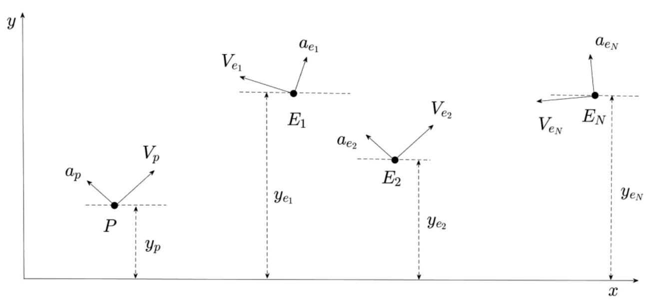

Fig. 2 Engagement geometry.

axis. Then the planar motions of the pursuer P and the evaders E_j, $j = 1, \ldots, N$, are described by nonlinear differential equations

$$\begin{aligned} \dot{x}_i &= V_i \cos\varphi_i, \\ \dot{y}_i &= V_i \sin\varphi_i, \quad i = p, \quad e_1, e_2, \ldots, e_N \\ \dot{\varphi}_i &= \frac{a_i}{V_i} \end{aligned} \tag{1}$$

The distances between the pursuer and the evaders are

$$r_j(t) = \sqrt{(x_{e_j}(t) - x_p(t))^2 + (y_{e_j}(t) - y_p(t))^2}, \quad j = 1, \ldots, N \tag{2}$$

In the pursuit of the evader E_j, the engagement final time (the moment of the minimal distance between the pursuer and E_j) is

$$t^n_{f_j} = \arg\min_{t \ge t_0}\{r_j(t)\dot{r}_j(t) \ge 0\}, \qquad j = 1, \ldots, N \tag{3}$$

The time remaining to the end of the pursuit (time-to-go) is approximated as

$$\hat{t}_{g_j} = -\frac{r_j}{\dot{r}_j}, \qquad j = 1, \ldots, N \tag{4}$$

The miss distance in the jth pursuit is defined as

$$\mathrm{MD}_j = r(t^n_{f_j}), \qquad j = 1, \ldots, N \tag{5}$$

Now, let us assume that the deviations of the pursuer and the evaders from the collision course are small during the engagement. This allows linearizing the relative trajectories along the nominal collision geometry [10] and calculating N engagement durations:

$$t_{f_j} = \frac{r_j(0)}{V_p \cos\varphi_p(0) - V_{e_j}\cos\varphi_{e_j}(0)}. \qquad j = 1, \ldots, N \tag{6}$$

By Eq. (6), the zero separation in the x direction between the pursuer and the evader E_j for $t = t_{f_j}$ is guaranteed. The separations in the y direction are

$$y_j = y_{e_j} - y_p, \qquad j = 1, \ldots, N \tag{7}$$

It is assumed that, for $t \in [t_0, \max_{j=1,\ldots,N} t_{f_j}]$, the controller dynamics of the pursuer is described by the linear differential equation

$$\dot{x}_p = A_p x_p + B_p u_p, \qquad x_p(t_0) = x_{p0} \tag{8}$$

$$a_p = C_p x_p + d_p u_p \tag{9}$$

where x_p is the state vector consisting of n_p internal variables, and u_p is the guidance command. For example, an ideal pursuer is characterized by $A_p = B_p = C_p = 0, d_p = 1, n_p = 0$, meaning that $a_p = u_p$. The pursuer with the first-order strictly proper dynamics is described by Eqs. (8) and (9), where the state variable x_p coincides with the pursuer's lateral acceleration a_p ($n_p = 1$, $A_p = -1/\tau_p$, $B_p = 1/\tau_p, C_p = 1, d_p = 0$), and τ_p is the controller time constant.

In this paper, all the evaders are assumed to have an ideal dynamics, meaning that their lateral accelerations $a_{e_j}(t)$ are identical to the acceleration commands $u_{e_j}(t)$, $j = 1, \ldots, N$. Moreover, the functions $u_{e_j}(t)$, $j = 1, \ldots, N$, are known to the pursuer.

Let us define N state vectors:

$$\begin{aligned} x_j &= [y_j, \dot{y}_j, x_p^T]^T \triangleq [x_1, x_2, x_3, \ldots, x_{n_p+2}]^T \in \mathbb{R}^{n_p+2}, \\ & j = 1, \ldots, N \end{aligned} \tag{10}$$

where y_j are the relative separations [Eq. (7)] and x_p is the pursuer's controller state vector in Eq. (8). Then, due to Eqs. (8) and (9) and the small angles assumption, the system dynamics is described by linear differential equations

$$\dot{x}_j = Ax + Bu_p + Cu_{e_j}, \qquad t \in [t_0, t_{f_j}], \qquad j = 1, \ldots, N \tag{11}$$

where

$$A = \begin{bmatrix} 0 & 1 & [0] \\ 0 & 0 & -C_p \\ [0] & [0] & A_p \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ -d_p \\ B_p \end{bmatrix}, \qquad C = \begin{bmatrix} 0 \\ 1 \\ [0] \end{bmatrix} \tag{12}$$

"[0]" denotes a zero matrix of appropriate dimension.

B. Minimum Control Effort Problem with Delayed Pursuit Decision

For $t = t_0$ the pursuer is not aware which evader of E_j, $j = 1, \ldots, N$, it should intercept. The decision is taken at the prescribed time moment $t_d > t_0$. It is known in advance that the probability to pursue the evader E_j is equal to $p_j > 0$, satisfying

$$\sum_{j=1}^{N} p_j = 1 \tag{13}$$

If $u_{p_j}(\cdot)$ denotes the control of the pursuer in the case that it engages evader E_j, then the average control effort over the set of evaders is

$$J = \sum_{j=1}^{N} p_j \int_{t_0}^{t_{f_j}} u^2_{p_j}(t)\,\mathrm{d}t \tag{14}$$

This cost function has to be minimized by choosing the inputs $u_{p_j}(\cdot)$, $j = 1, \ldots, N$, under the following constraints:

1) The miss with respect to the evader E_j when using u_{p_j} is zero:

$$y_j(t_{f_j}) = 0, \qquad j = 1, \ldots, N \tag{15}$$

2) The N inputs coincide before the decision time t_d:

$$u_{p_1}(t) = u_{p_2}(t) = \ldots = u_{p_N}(t) = u_p(t), \qquad t \in [t_0, t_d] \tag{16}$$

III. Solution

A. Model Reduction Using Zero Effort Transformation

Let us use the scalarizing transformation [11]

$$z_j = D\left(\Phi(t_{f_j}, t)x_j + \int_t^{t_{f_j}} \Phi(t_{f_j}, \tau)Cu_{e_j}(\tau)\,\mathrm{d}\tau\right) \tag{17}$$

where $D_j = (1, 0, [0]_{1\times n_p})$, and $\Phi(t,\tau)$ is the transition matrix of a homogeneous system associated with Eq. (11). Then the zero-effort miss (ZEM) z_j satisfies the scalar differential equation

$$\dot{z}_j = h_j(t)u_{p_j}, \qquad t\in[t_0, t_{f_j}], \qquad j = 1,\dots,N \tag{18}$$

where

$$h_j(t) = D\Phi(t_{f_j}, t)B \tag{19}$$

B. After the Decision Moment

Assume that at $t = t_d$, the pursuer decides to pursue the jth evader. Then, for $t\in[t_d, t_{f_j}]$, the problem is to minimize the control effort

$$J_j = \int_{t_d}^{t_{f_j}} u_{p_j}^2(t)\,\mathrm{d}t, \qquad j = 1,\dots,N \tag{20}$$

subject to the scalar differential equations (18) with $z_j(t_{f_j}) = 0$. Let us denote

$$z_j(t_d) = z_{j_d}, \qquad j = 1,\dots,N \tag{21}$$

Then, by the Pontryagin's maximum principle [12], the open-loop optimal acceleration commands are produced by the linear optimal guidance laws [13]:

$$u_{p_j}^* = c_j h_j(t) z_{j_d}, \qquad j = 1,\dots,N \tag{22}$$

where

$$c_j = \frac{1}{\int_{t_d}^{t_{f_j}} h_j^2(t)\,\mathrm{d}t} \tag{23}$$

This yields the optimal costs:

$$J_j^* = c_j z_{j_d}^2, \qquad j = 1,\dots,N \tag{24}$$

C. Before the Decision Moment

The cost functional (14) can be rewritten as

$$J = \int_{t_0}^{t_d} u_p^2(t)\,\mathrm{d}t + \sum_{j=1}^N p_j J_j^* \tag{25}$$

Because of Eq. (24),

$$J = \sum_{j=1}^N p_j c_j z_{j_d}^2 + \int_{t_0}^{t_d} u_p^2(t)\,\mathrm{d}t \tag{26}$$

where, due to Eq. (18),

$$z_{j_d} = z_{j_0} + \int_{t_0}^{t_d} h_j(t)u_p(t)\,\mathrm{d}t, \qquad j = 1,\dots,N \tag{27}$$

Let us introduce the vectors

$$z_d = \begin{bmatrix} z_{1_d} \\ \vdots \\ z_{N_d} \end{bmatrix}, \qquad z_0 = \begin{bmatrix} z_{1_0} \\ \vdots \\ z_{N_0} \end{bmatrix}, \qquad h(t) = \begin{bmatrix} h_1(t) \\ \vdots \\ h_N(t) \end{bmatrix} \tag{28}$$

and the diagonal matrix

$$P = \mathrm{diag}\left\{\frac{1}{p_1c_1},\dots,\frac{1}{p_Nc_N}\right\} \tag{29}$$

Then the cost functional (26) is rewritten as

$$J = z_d^T P^{-1} z_d + \int_{t_0}^{t_d} u_p^2(t)\,\mathrm{d}t \tag{30}$$

where

$$z_d = z_0 + \int_{t_0}^{t_d} h(t)u_p(t)\,\mathrm{d}t \tag{31}$$

Remark 1: By direct calculation, it can be shown that for any vector $z\in\mathbb{R}^N$ and for any symmetric positive-definite $N\times N$-matrix P,

$$z^T P^{-1} z = \max_{l\in\mathbb{R}^N}\left[l^T z - \frac{1}{4} l^T P l\right] \tag{32}$$

By using Eq. (32) for $z = z_d$ and taking into account Eq. (31), the cost functional is represented as

$$J = \max_{l\in\mathbb{R}^N} f(u_p(\cdot), l) \tag{33}$$

where

$$f(u_p(\cdot), l) = l^T z_0 - \frac{1}{4} l^T P l + \int_{t_0}^{t_d} [u_p^2(t) + (l^T h(t))u_p(t)]\,\mathrm{d}t \tag{34}$$

Remark 2: It can be shown that the maximum over $\mathbb{R}^N$ in Eq. (33) can be relaxed to

$$J = \max_{\|l\|\le L} f(u_p(\cdot), l) \tag{35}$$

where $L > 0$ is a sufficiently large number. Thus, by applying Corollary 3.3 in [14],

$$\min_{u_p(\cdot)}\max_{l\in\mathbb{R}^N} f(u_p(\cdot), l) = \max_{l\in\mathbb{R}^N}\min_{u_p(\cdot)} f(u_p(\cdot), l) \tag{36}$$

Because of Eq. (36),

$$\min_{u_p(\cdot)} J = \max_{l\in\mathbb{R}^N}\min_{u_p(\cdot)} f(a(\cdot), l) \tag{37}$$

Let us calculate the maximin in the right-hand side of Eq. (37). For any fixed $l\in\mathbb{R}^N$, the minimizing function is

$$u_p^*(t) = u_p^*(t; l) = -\frac{1}{2} l^T h(t) \tag{38}$$

By substituting Eq. (38) into Eq. (34),

$$\psi(l) \triangleq f(u_p^*(\cdot; l), l) = l^T z_0 - \frac{1}{4} l^T (P + G(t_0)) l \tag{39}$$

where the symmetric matrix $G(t)$ is

$$G(t) = \int_t^{t_d} h(\xi)h^T(\xi)\,\mathrm{d}\xi \tag{40}$$

Note that $G(t) \ge 0$, which along with Eq. (29) guarantees that the matrix $P + G(t_0)$ is positive definite and, in particular, invertible. The maximizing vector in the right-hand side of Eq. (37) is

$$l^* = 2(P + G(t_0))^{-1} z_0 \tag{41}$$

By substituting Eq. (41) into Eq. (38),

$$u_p^*(t) = u_p^*(t, l^*) = -h^T(t)(P + G(t_0))^{-1} z_0 \tag{42}$$

By using Eq. (42) and by replacing (t_0, z_0) with (t, z), where $z = (z_1, z_2, \ldots, z_N)^T$, the optimal feedback is obtained as

$$u_p^*(t, z) = K(t) z \tag{43}$$

where

$$K(t) = (K_1(t), K_2(t), \ldots, K_N(t)) = -h^T(t)(P + G(t))^{-1} \tag{44}$$

and the matrices P and $G(t)$ are given by Eqs. (29) and (40), respectively.

Let us define the times-to-go

$$t_{g_j} \triangleq t_{f_j} - t, \qquad j = 1, \ldots, N \tag{45}$$

Remark 3: In the particular case where the pursuer has an ideal dynamics and the evaders have constant accelerations $a_{e_j}(t) \equiv a_{e_j} = \text{const}$, $j = 1, \ldots, N$, the zero-effort miss is

$$z_j = t_{g_j}^2 \left(V_{c_j} \dot{\lambda}_j + \frac{1}{2} a_{e_j} \right), \qquad j = 1, \ldots, N \tag{46}$$

where $V_{c_j} = V_p + V_{e_j}$ is the closing speed between the pursuer and the jth evader, and λ_j is the line-of-sight angle between the pursuer and the jth evader. Then, by virtue of Eq. (44), the optimal feedback (43) in this case is represented as a linear combination of the augmented proportional navigation (APN) guidance algorithms with time-varying coefficients to each of the evaders:

$$u_p^*(t, z) = \sum_{j=1}^{N} N_j(t) \left(V_{c_j} \dot{\lambda}_j + \frac{1}{2} a_{e_j} \right) \tag{47}$$

where

$$N_j(t) = K_j(t) t_{g_j}^2, \qquad j = 1, \ldots, N \tag{48}$$

D. Particular Cases

1. Pursuer with Ideal Dynamics

Consider the case where the pursuer has an ideal dynamics, that is, in Eqs. (8) and (9), $A_p = B_p = C_p = 0$, $d_p = 1$, $n_p = 0$, and, respectively, $a_p = u_p$. In this case,

$$h_j(t) = -(t_{f_j} - t), \qquad j = 1, \ldots, N \tag{49}$$

Define the quantities

$$t_{g_d} \triangleq t_d - t, \qquad t_{gd_j} \triangleq t_{f_j} - t_d, \qquad j = 1, \ldots, N \tag{50}$$

Then, the coefficients c_j given by Eq. (23) are

$$c_j = \frac{1}{\int_{t_d}^{t_{f_j}} (t_{f_j} - t)^2 \, \mathrm{d}t} = \frac{3}{t_{gd_j}^3}, \qquad j = 1, \ldots, N \tag{51}$$

yielding

$$P = \operatorname{diag}\left\{ \frac{t_{gd_j}^3}{3 p_j} \right\}, \qquad j = 1, \ldots, N \tag{52}$$

Because of Eqs. (40) and (49), the elements of the matrix $G(t)$ are

$$G_{ij}(t) = \int_t^{t_d} (t_{f_i} - \xi)(t_{f_j} - \xi) \, \mathrm{d}\xi = \frac{1}{3} t_{g_d}^3 + \frac{1}{2} t_{g_d} (t_{g_i} t_{gd_j} + t_{g_j} t_{gd_j}), \quad i, j = 1, \ldots, N \tag{53}$$

For $N = 2$, we denote $p_1 = p$, $p_2 = 1 - p$. In this case, the gains (48) are calculated explicitly:

$$N_1 = N_1(t_{g_1}, t_{g_2}, t_{gd_1}, t_{gd_2}, t_{gd}) = \frac{1}{A} (A_3 t_{g_1} - A_2 t_{g_2}) t_{g_1}^2 \tag{54}$$

$$N_2 = N_2(t_{g_1}, t_{g_2}, t_{gd_1}, t_{gd_2}, t_{gd}) = \frac{1}{A} (-A_2 t_{g_1} + A_1 t_{g_2}) t_{g_2}^2 \tag{55}$$

where

$$A_1 = G_{11} + P_{11} = \frac{1}{p c_1} \frac{t_{g_1}^3 - t_{gd_1}^3}{3} + \frac{1}{3p} t_{gd_1}^3, \qquad A_2 = G_{12} \tag{56}$$

$$A_2 = \frac{1}{3} t_{gd}^3 + \frac{1}{2} t_{gd} (t_{g_1} t_{gd_2} + t_{g_2} t_{gd_1}) \tag{57}$$

$$A_3 = G_{22} + P_{22} = \frac{t_{g_2}^3 - t_{gd_2}^3}{3} + \frac{1}{3(1-p)} t_{gd_2}^3 \tag{58}$$

$$A = \det(P + G(t)) = A_1 A_3 - A_2^2 \tag{59}$$

For $N > 2$, the gains (48) also can be represented explicitly in terms of the time-to-go (45) and quantities (50). For the sake of brevity, we do not present these expressions.

2. Pursuer with First-Order Strictly Proper Dynamics

The pursuer with the first-order strictly proper dynamics is described by Eqs. (8) and (9) with $A_p = -1/\tau_p$, $B_p = 1/\tau_p$, $C_p = 1$, $d_p = 0$, $n_p = 1$, where the state variable x_p coincides with the pursuer's lateral acceleration a_p, and τ_p is the controller time constant. In this case,

$$h_j(t) = -\tau_p (\exp[-(t_{f_j} - t)/\tau_p] + (t_{f_j} - t)/\tau_p - 1), \quad j = 1, \ldots, N \tag{60}$$

Note that in this case, the gains (48) also can be represented explicitly in terms of the time-to-go (45) and quantities (50), but the expressions are more bulky.

IV. Simulation Results

A. Two Evaders

Numerical simulations were carried out in the case of the pursuer with the ideal dynamics and two diverging evaders for the parameters presented in Table 1. For these parameters, $t_{f_1} = 4.5$ s, $t_{f_2} = 6.25$ s.

Notice that, although the decision taken by the pursuer at time t_d is probabilistic, we can obtain the performance of the guidance law without Monte Carlo simulations. Indeed, by simulating both cases, that is, the pursuer goes after E_1 and after E_2, respectively, all statistical performance indicators can be obtained by averaging over the evader set, using the known probability of each decision result.

1. Linear Simulation

In this section, the optimal pursuer's strategy (43) is applied to the linearized systems (11).

Table 1 Two-evader simulation parameters

Parameter	Symbol	Value	Unit
Speed E_1	V_{e_1}	400	m/s
Initial position E_1	$(x_{0_{e_1}}, y_{0_{e_1}})$	(3600, 0)	m
Lateral acceleration E_1	a_{e_1}	30	m/s^2
Speed E_2	V_{e_2}	400	m/s
Initial position E_2	$(x_{0_{e_2}}, y_{0_{e_2}})$	(5000, 0)	m
Lateral acceleration E_2	a_{e_2}	−30	m/s^2
Speed P	V_p	400	m/s
Initial position P	(x_{0_p}, y_{0_p})	(0, 0)	m
Decision time	t_d	3	s

In Fig. 3, the optimal trajectories of the pursuer are shown in the cases of the homing to the evader E_1 or to the evader E_2 for different values of the probability p. It is seen that for $t \in [t_0, t_d]$, the pursuer's trajectory lies "between" the evaders and "closer" to E_1 for larger p. For $t \in [t_d, t_f]$ (after the decision), the pursuer's trajectories are shaped differently, making the homing on E_1 easier for a larger probability p.

In Fig. 4, the respective optimal acceleration profiles of the pursuer are shown. If the decision was to intercept E_1, then for larger values of p the acceleration effort is lower. For the opposite decision, the dependence is opposite.

Because of Eqs. (55–59), it can be shown that for $p = 0$, $N_1 = 0$ and $N_2 = 3$ for any values of $t_{g_1}, t_{g_2}, t_{gd_1}, t_{gd_2}, t_{gd}$. In this case, the optimal strategy becomes APN against the second evader. Correspondingly, for $p = 1$, $N_1 = 3$ and $N_2 = 0$, yielding APN against the first evader. The gain N_1 decreases and the gain N_2 increases as functions of $p \in [0, 1]$. These facts are illustrated in Fig. 5, depicting the gains N_1 and N_2 as functions of p for $t = 0.5t_d = 1.5$ s.

In Fig. 6, the performance of the optimal control strategy Eqs. (47) and (48) is compared with that of two nonoptimal strategies, not taking into account that at $t = t_d$ the pursuer decides to pursue the evader E_1 with the probability p or the evader E_2 with the probability $1 - p$. The two strategies for comparison are the APN against E_1 and against E_2 for $t \in [t_0, t_d]$. In Fig. 6, the average of the control effort $J = pJ_1 + (1 - p)J_2$ is shown as a function of p. It is seen that the optimal control guarantees the smallest values of J for $p \in (0, 1)$ when compared with the alternative strategies of just homing on one of the evaders, not taking into account that later in flight the other evader may need to be engaged. For $p = 0$, the optimal strategy is the APN against E_2, whereas for $p = 1$, it is the APN against E_1, but between these extreme values there is a lot to save by choosing the proposed guidance law.

2. *Nonlinear Simulation*

In this section, the performance of the optimal strategy (43) is verified in a more realistic nonlinear simulation; that is, the optimal pursuer's strategy (43) is applied to the original nonlinear systems (1). Our main purpose in this section is to show that the linearization assumption that we made in order to derive the guidance law is adequate. The nonlinear simulation of the pursuit of E_j is carried out

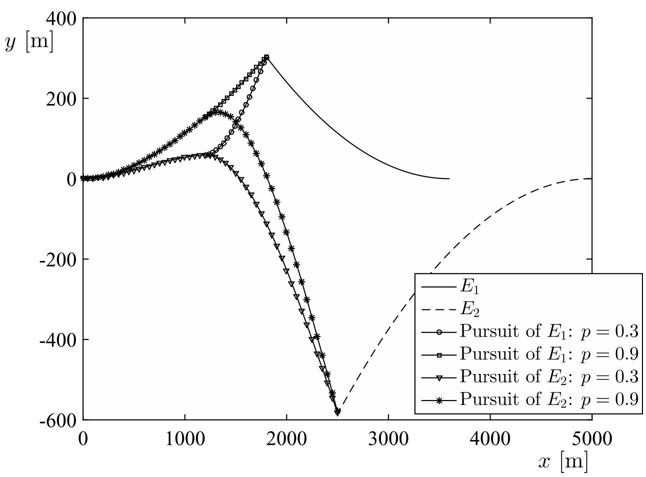

Fig. 3 Linear simulation: optimal pursuer's trajectories and evader's trajectories.

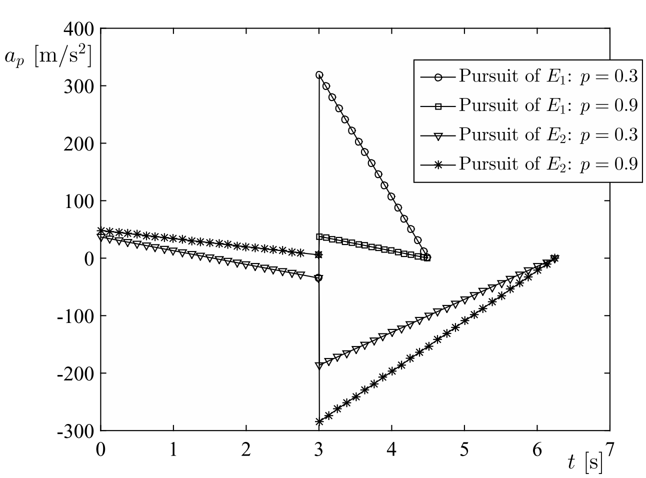

Fig. 4 Linear simulation: pursuer's optimal acceleration profiles.

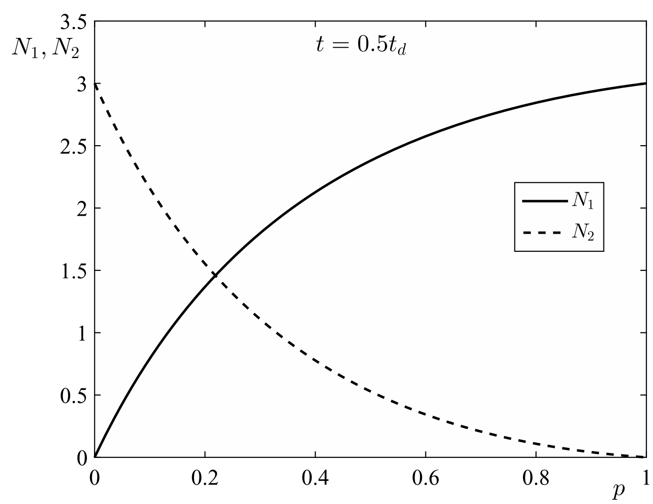

Fig. 5 Optimal gains for $t = 0.5t_d$.

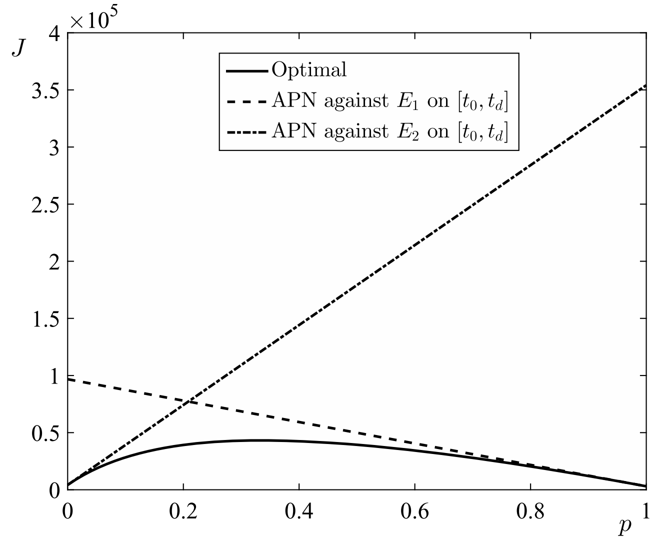

Fig. 6 Average control effort.

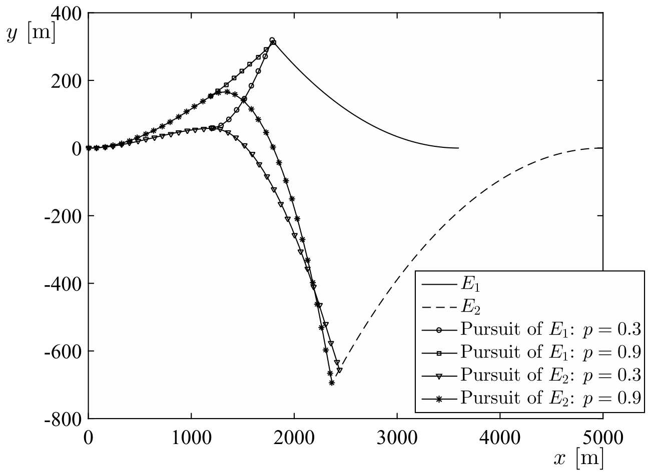

Fig. 7 Nonlinear simulation: optimal pursuer's trajectories and evader's trajectories.

for $t \leq \hat{t}_{f_j}$, where $\hat{t}_{f_j}$ is given by Eq. (3). The time-to-go is estimated at each time step according to Eq. (4).

Figure 7 is a counterpart of Fig. 3 for the nonlinear simulation. In this simulation, $\varphi_p(0) = \varphi_{e_1}(0) = \varphi_{e_2}(0) = 0$ deg. It is seen that the trajectories are similar to those in the linear simulation. Figure 8, where the trajectories obtained in the linear and nonlinear simulation are compared for the case of pursuit of E_1 with $p = 0.3$, demonstrates closeness of the trajectories in the linear and nonlinear simulation.

The optimal acceleration profiles for $p = 0.3$ are compared in Fig. 9 for the pursuit of E_1 and E_2. It is seen that the acceleration profiles in the nonlinear simulation are close to those in the linear simulation.

Figure 10 is a nonlinear version of Fig. 6. It is seen that the average control effort for the optimal strategy and for two nonoptimal controls depends on p very similarly to a linear case.

Let us consider the realistic case where the lateral acceleration (control) of the pursuer is bounded $|a_p| \leq a_p^{\max}$. In this case, zero miss distance cannot be guaranteed by any pursuit strategy. Define the average miss distance

$$\mathrm{MD}_{\mathrm{ave}} = p\mathrm{MD}_1 + (1 - p)\mathrm{MD}_2 \tag{61}$$

where the miss distances MD_j, $j = 1, 2$, are given by Eq. (5).

In Fig. 11, the average miss distances are depicted as functions of p for three strategies: optimal strategy [Eqs. (47) and (48)] and two nonoptimal strategies using APN control against one of the evaders for $t \in [t_0, t_d]$ and eventually switching to the other evader with the given probability p, respectively $1 - p$. These results were obtained for $a_p^{\max} = 220$ m/s^2, $x_{e_2} = 4000$ m, $t_d = 2.8$ s, and other parameters coinciding with those of Table 1. The graphs for nonoptimal strategies are cut at the level of 20 m. It is seen that the optimal guidance that takes the probability p into account provides small average miss distances (smaller than 50 cm) for $p \in [0.35, 0.75]$, whereas the average miss distances obtained using the nonoptimal strategies are large in the whole probability range, except when p is small or very close to 1; that is, there is almost no uncertainty. However, the optimal guidance law can handle this uncertainty very well.

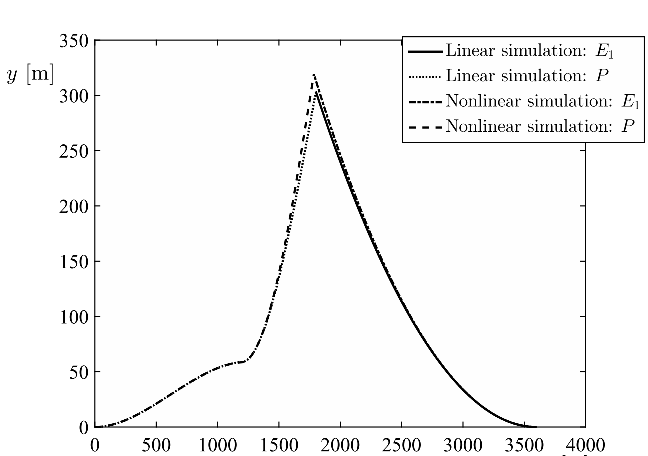

Fig. 8 Trajectories: linear vs nonlinear simulation.

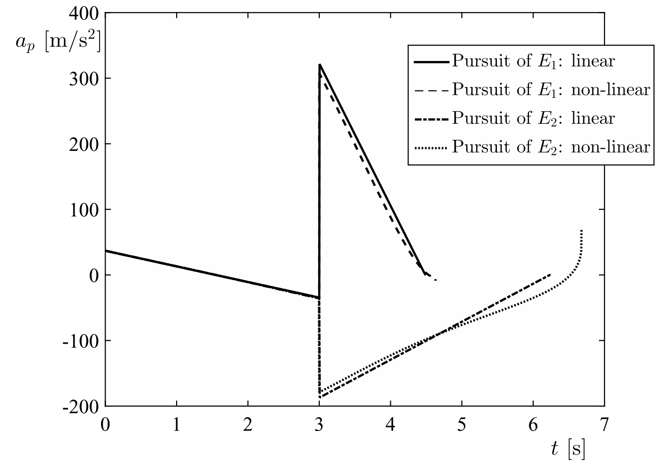

Fig. 9 Optimal acceleration profiles: nonlinear vs linear simulation.

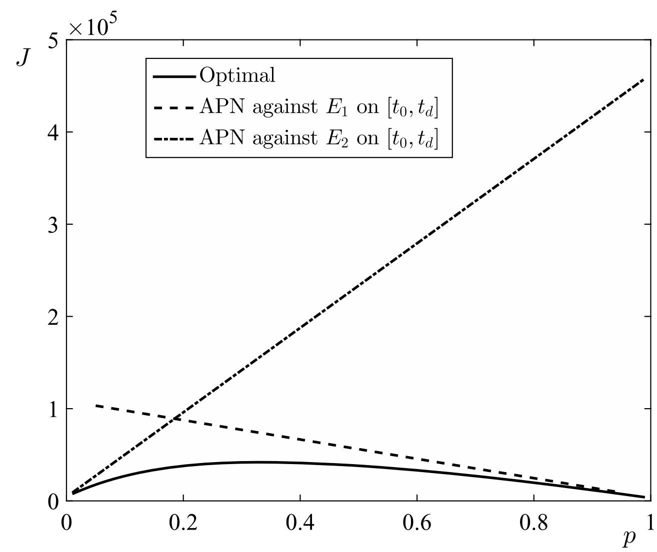

Fig. 10 Average control effort in nonlinear simulation.

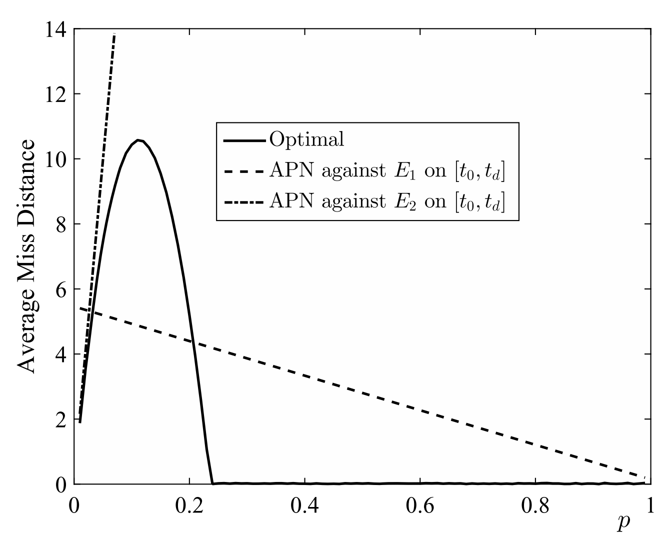

Fig. 11 Average miss distance for bounded control.

Table 2 Three-evader simulation parameters

Parameter	Symbol	Value	Unit
Speed E_1	V_{e_1}	400	m/s
Initial position E_1	$(x_{0_{e_1}}, y_{0_{e_1}})$	(3600, 0)	m
Lateral acceleration E_1	a_{e_1}	30	m/s^2
Speed E_2	V_{e_2}	400	m/s
Initial position E_2	$(x_{0_{e_2}}, y_{0_{e_2}})$	(4200, 0)	m
Lateral acceleration E_2	a_{e_2}	0	m/s^2
Speed E_3	V_{e_3}	400	m/s
Initial position E_3	$(x_{0_{e_3}}, y_{0_{e_3}})$	(5000, 0)	M
Lateral acceleration E_3	a_{e_3}	−30	m/s^2
Speed P	V_p	400	m/s
Initial position P	(x_{0_p}, y_{0_p})	(0, 0)	m
Decision time	t_d	3	s

B. Three Evaders

In this section, the results of the nonlinear simulation for the case of three evaders are presented. The first and the third evaders are the same as in the two-target simulation; the second target is nonmaneuverable (see the simulation parameters in Table 2). In Fig. 12, the optimal pursuer's trajectories and the evader's trajectories are shown for $p_1 = 0.3$, $p_2 = 0.2$, and $p_3 = 0.5$. In Fig. 13, the average control efforts for the optimal strategy and for three nonoptimal controls are depicted for fixed probability $p_1 = 0.2$ as functions of p_2. It is seen that the optimal strategy performs better than others, as in the two-evader simulation.

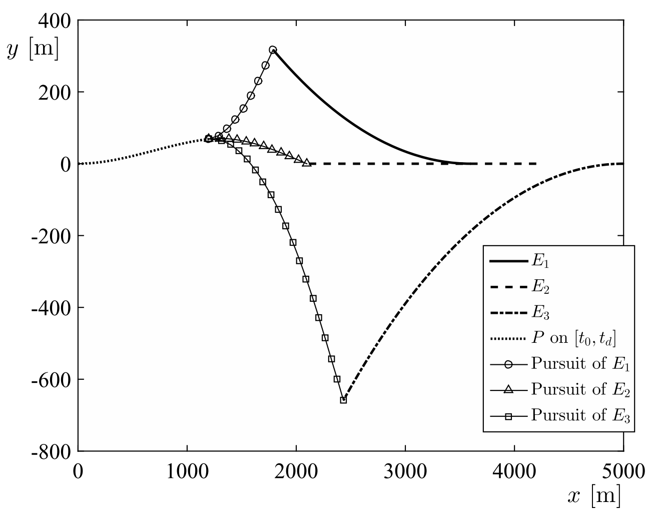

Fig. 12 Three evaders: optimal pursuer's trajectories and evader's trajectories.

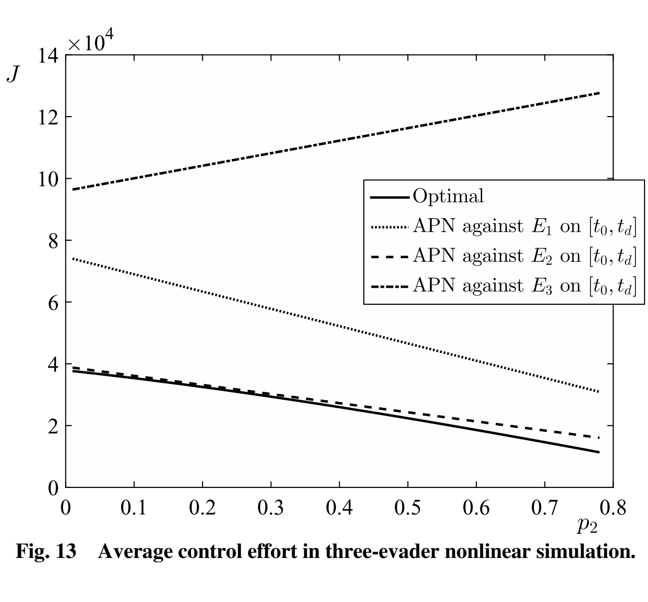

Fig. 13 Average control effort in three-evader nonlinear simulation.

V. Conclusions

A minimum control effort pursuit strategy was proposed for the case in which the pursuer is confronted with multiple evaders and the decision which of the evaders should be engaged is taken only later in the flight and the probability of each decision outcome is assumed to be known. The strategy is a weighted sum of optimal guidance commands, each corresponding to the engagement of one of the evaders, and so the structure of the algorithm is very simple. However, the expressions of the weighting coefficients are complex as they depend on the time-to-go to each of the evaders and the outcome probabilities of the engagement decision. In this paper, it is assumed that these probabilities remain constant until the decision time. In practice, the probabilities may change during the flight as more information becomes available. The consequences of this fact will be explored as part of future work.

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