Novel Method and Experimental Validation of Statistical Calibration via Gaussianization in Hot-Wire Anemometry

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69th Annual Meeting of the APS Division of Fluid Dynamics

DEPARTMENT OF AEROSPACE ENGINEERING | TECHNION Israel Institute of Technology

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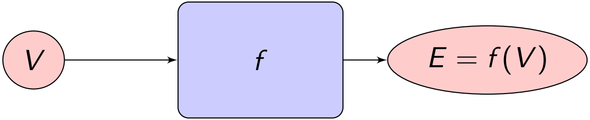
Sensor I/O Mapping



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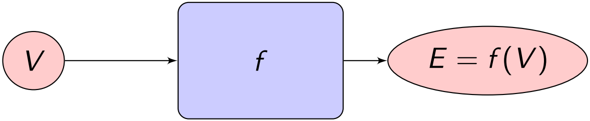

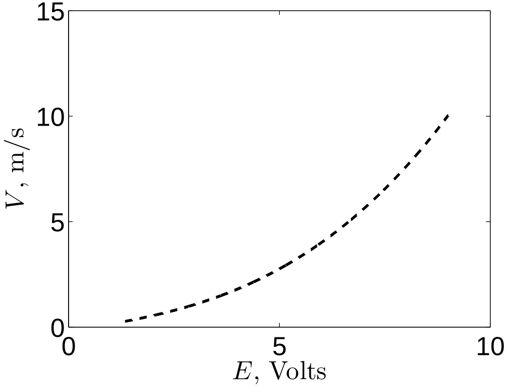

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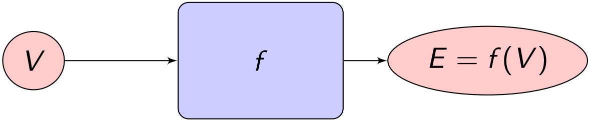

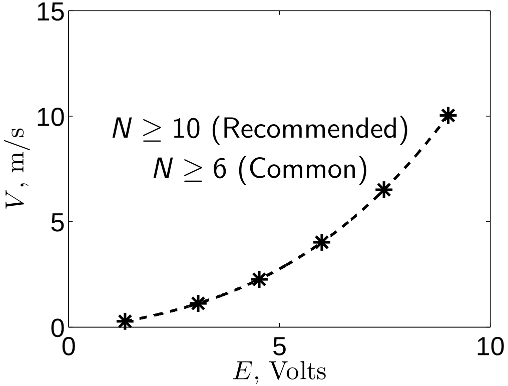

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Objectives and Motivation

Obtain efficient calibration technique

- Only $N = 2$ calibration data points.
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Statistical Properties of Turbulent Flow

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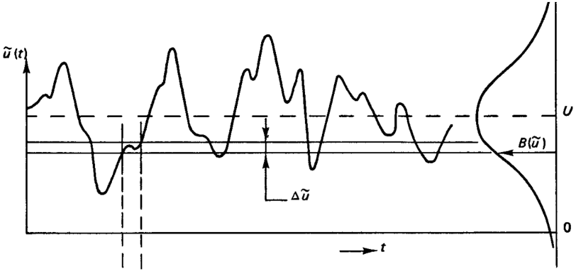

[Tennekes & Lumley, 1972]

Application of central-limit theorem (CLT)

$$V \sim N(\mu, \sigma)$$

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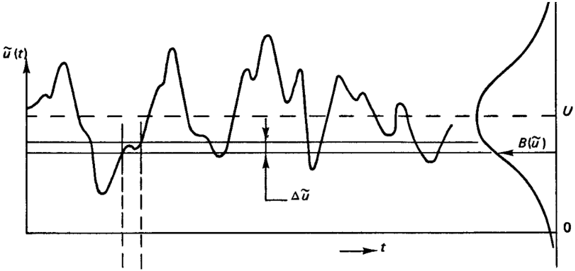

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Application of central-limit theorem (CLT)

$$V \sim N(\mu, \sigma)$$

Mathematical Foundation

X, F_X (cdf). Then, $U = F_X(X)$ is uniformly distributed on $[0, 1]$.

$$X = F_X^{-1}(U) \Rightarrow X, F_X$$

$\Phi(u)$-cdf of standard Gaussian r.v (the Laplace function).

$$Z \triangleq \Phi^{-1}(F_X(X)) \Rightarrow Z \sim N(0, 1)$$

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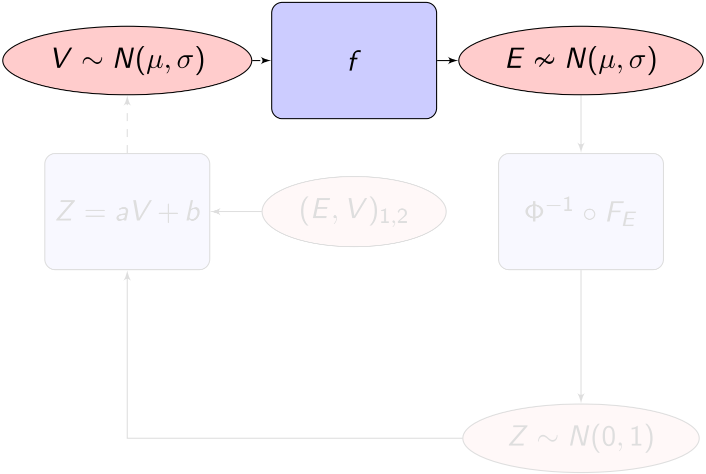
Calibration Procedure: Method of Gaussianization



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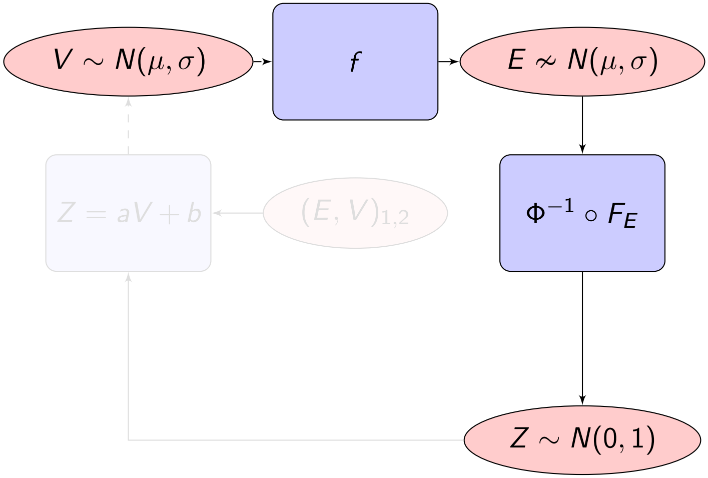

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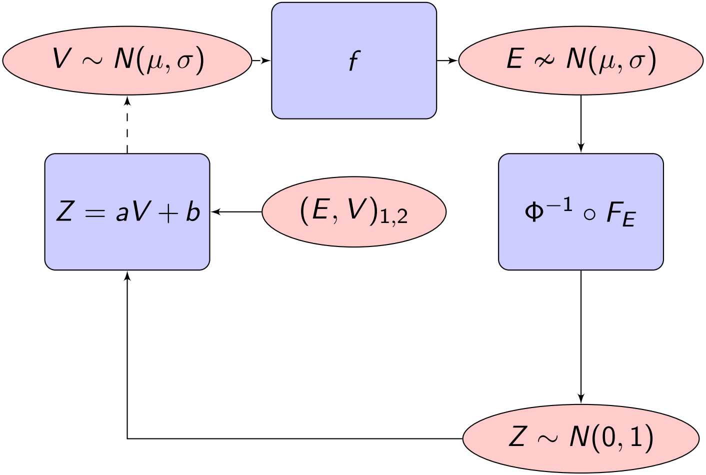

Desired Signal Properties

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- Should be as close to Gaussian as possible.
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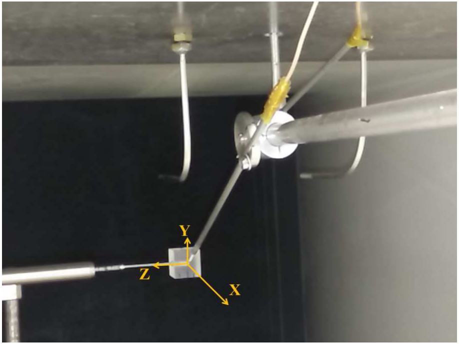


a_c = 20 mm. U = 5,7,9 m/s. i.e., Re$=\frac{a_c U}{\nu}$=6,400–11,500

Wake statistical properties

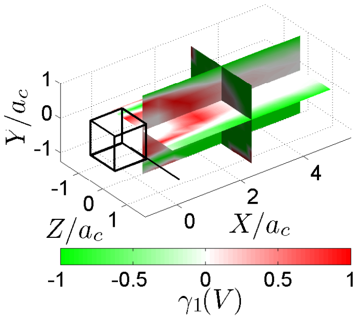
$$\mu_X = \mathbf{E}\left[X(t)\right] \qquad \sigma_V = \left[\mathbf{E}\left(X - \mu_X\right)^2\right]^{0.5}$$

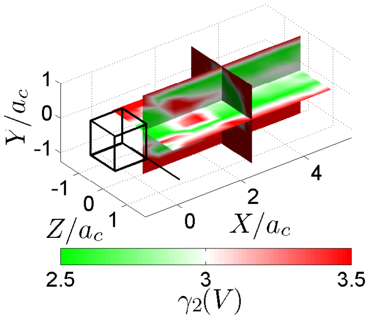
$$\gamma_1(X) = \mathbf{E}\left[\left(X - \mu_X\right)^3\right]/\sigma^3 \qquad \gamma_2(X) = \mathbf{E}\left[\left(X - \mu_X\right)^4\right]/\sigma^4$$

For Gaussian signal: $\gamma_1 = 0$ and $\gamma_2 = 3$

Wake statistical properties

Re=6,400 (U=5 m/s)





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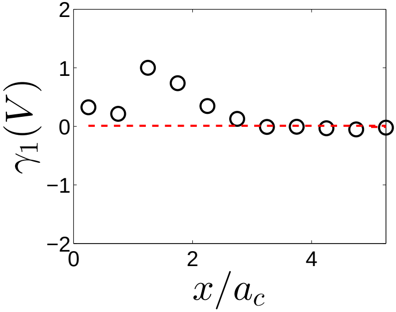
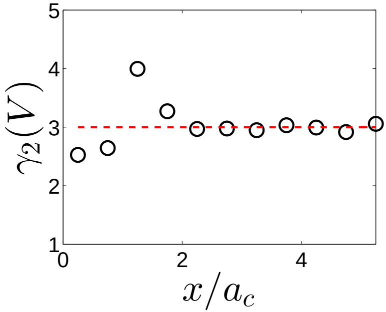
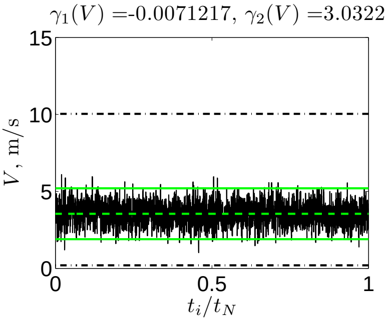
Re=6,400 (U=5 m/s) $y = 0.25a_c$, $z = -0.25a_c$



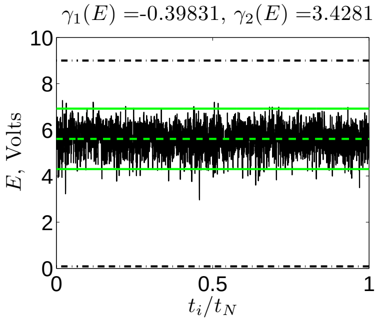


Application of the method

$$x = 3.75a_c,\ y = 0.25a_c,\ z = -0.25a_c$$

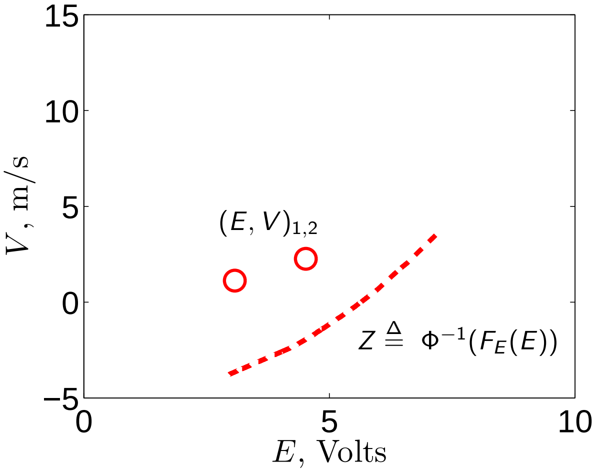


$\mu_V \pm 2.5\sigma_V$



$\mu_E \pm 2.5\sigma_E$

Validation



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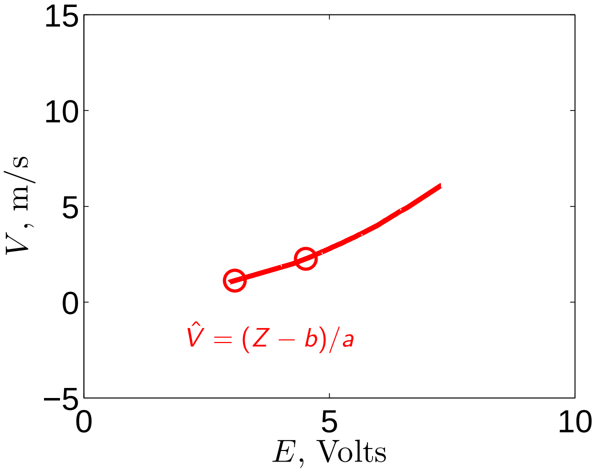

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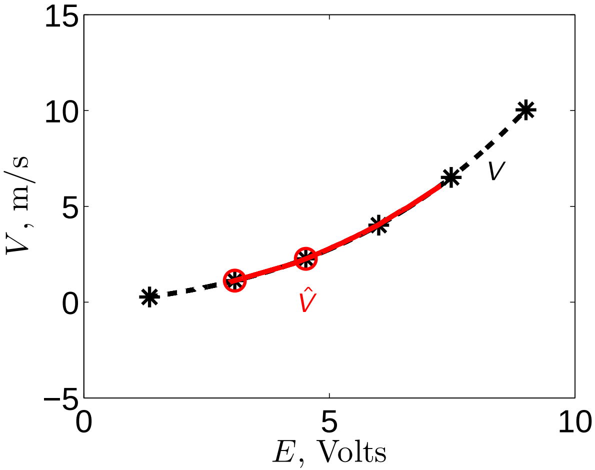

Conclusions

- Statistical method for hot-wire sensor calibration.
- Based on Gaussianization.
- Requires only two calibration data points.
- Can provide an extended calibration range.
- Performs well in $\mu_V \pm 2.5\sigma_V$ range of the signal $V(t)$.
- Performs well even for signals that are only approximately Gaussian.
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Acknowledgments

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(Technion Aerospace Turbulence Lab Research Engineers)

Mr. Mark Koifman
(Technion Wind Tunnel Complex Chief Engineer)

Mr. Nadav Shefer
(Technion Wind Tunnel Complex Technician)

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Hot-Wire Anemometer

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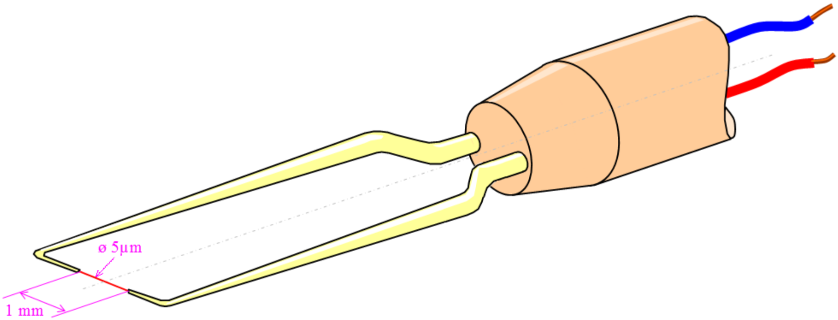
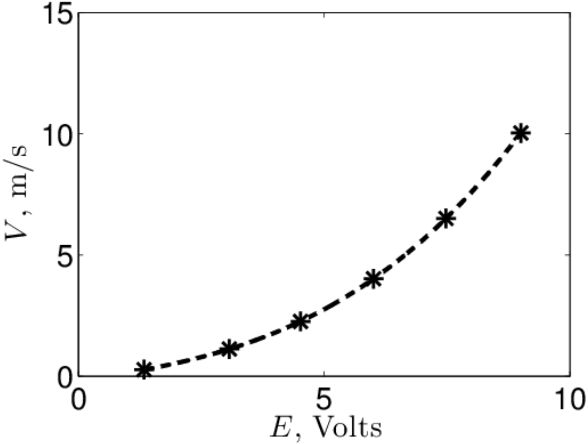


Image source:
`https://upload.wikimedia.org/wikipedia/commons/d/d3/Anemometre_a_fil_chaud%2C_hot-wire_anemometer.png`

Experimental Validation

- Tungsten wire (5 μm diameter; 1 mm length).
- CTA configuration.
- Freestream ($1 - 10\ m/sec$) of the wind tunnel; Pitot-static tube.



Method of Gaussianization

Consider the random variable

$$Z \stackrel{\Delta}{=} \Phi^{-1}(F_E(E))$$

$$\Downarrow$$

$$Z \sim N(0,1)$$

$$\Downarrow$$

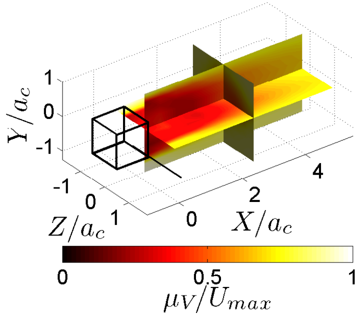
$$Z = aV + b$$

a, b are determined by two calibration data points.

Wake statistical properties

Re=6,400 (U=5 m/s)

$$\mu_X = E\left[X(t)\right]$$



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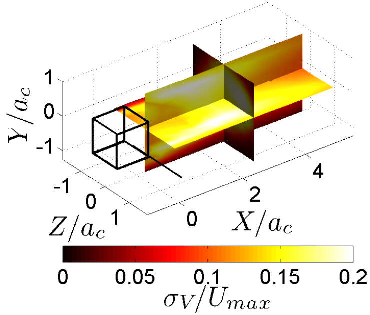

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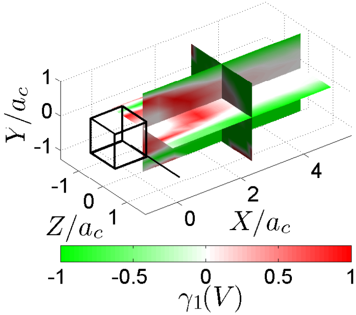

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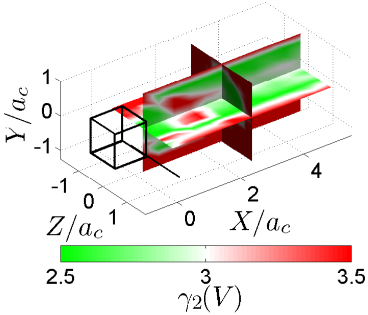

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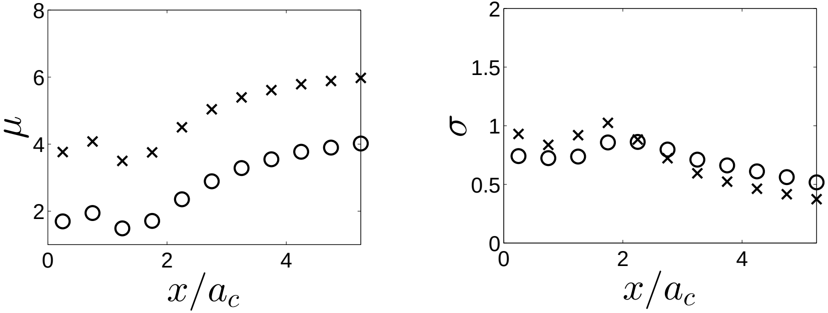

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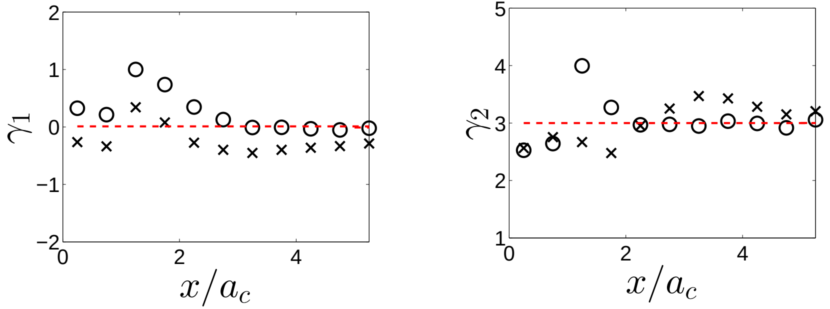

o - V   × - E

Estimation Error between V and $\hat{V}$

SE_i - local square error,

$$SE_i = [g(E_i) - \hat{g}(E_i)]^2, \;\; i = l, ..., m,$$

$$E_l = min[E(t)], \;\; E_m = max[E(t)].$$

MSE-mean-squared error,

$$MSE = \frac{1}{m - l + 1} \sum_{i=l}^{m} SE_i.$$

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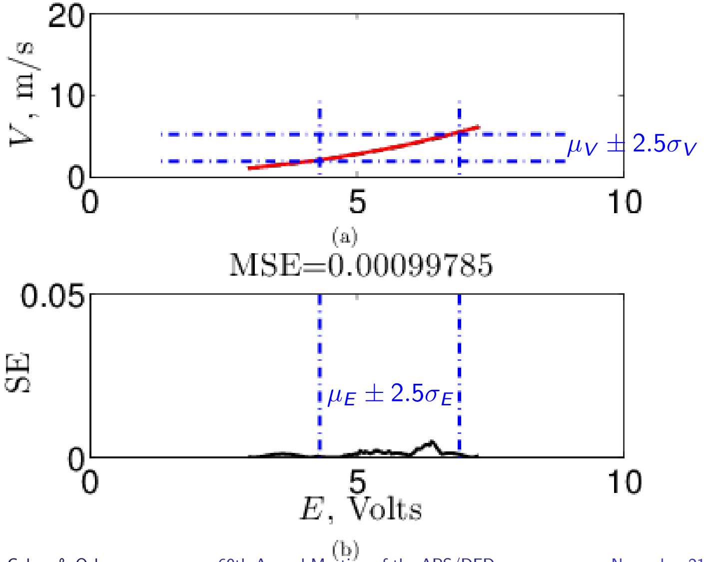
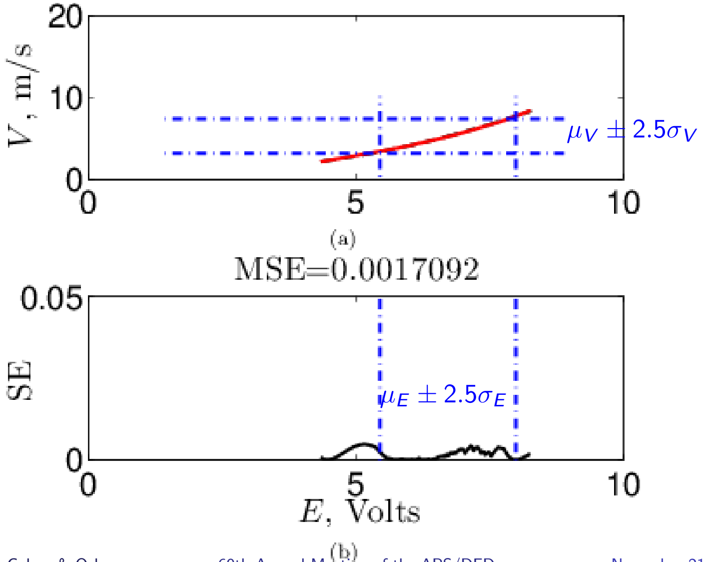


Estimation Error between V and $\hat{V}$

Re=8,900 (U= 7 m/s)



Estimation Error between V and $\hat{V}$

Re=11,500 (U= 9 m/s)