Statistical Calibration Via Gaussianization in Hot-Wire Anemometry

  

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TECHNION Israel Institute of Technology

Hot-Wire Anemometer

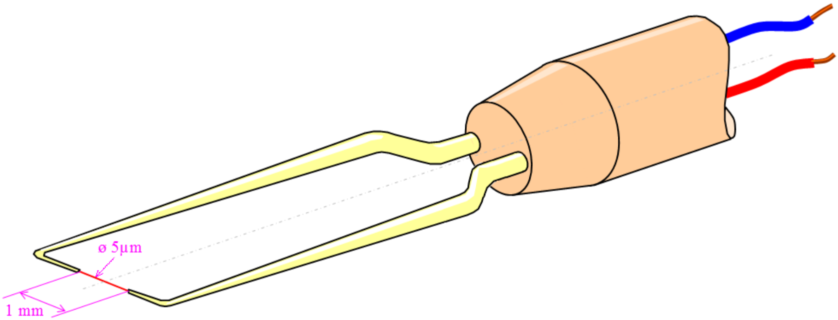
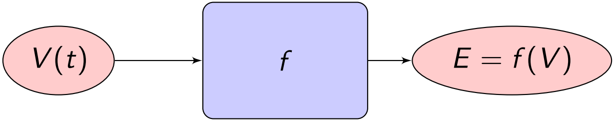


Image source:
https://upload.wikimedia.org/wikipedia/commons/d/d3/Anemometre_a_fil_chaud%2C_hot-wire_anemometer.png

Sensor I/O Mapping



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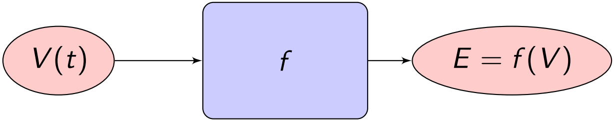
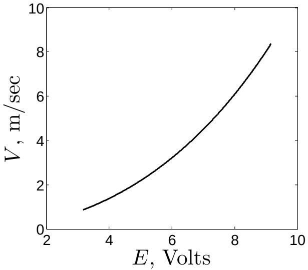


$$V = g(E)$$

$g = f^{-1}$- nonlinear mapping function



Hot-Wire Calibration Techniques

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- Polynomial fitting (4th order)

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- King's law (power law fitting)

$$E_i^2 = A + B \cdot V_i^n,\ 0.45 \leq n \leq 0.52, i = 1, .., N. \tag{2}$$

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$N \geq 7$ data points throughout the desired velocity range.

Objectives and Motivation

The Goals

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Obtain efficient calibration technique.

- Method to obtain $g = f^{-1}$ from sensor output E.
- Method that requires only $N = 2$ calibrated data points. Can save time.
- Method that acquires a wider velocity range extending out of the limits of the provided calibrated data.

Overview

Mathematical Foundation

Consider the random variable X, having F_X as its cdf. Then, the random variable $U \triangleq F_X(X)$ is uniformly distributed on $[0, 1]$.[Rohatgi, 1976]

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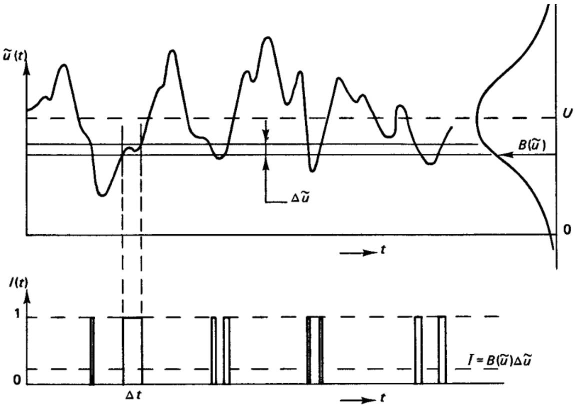
Statistical Properties of Turbulent Flow



[Tennekes & Lumley, 1972]

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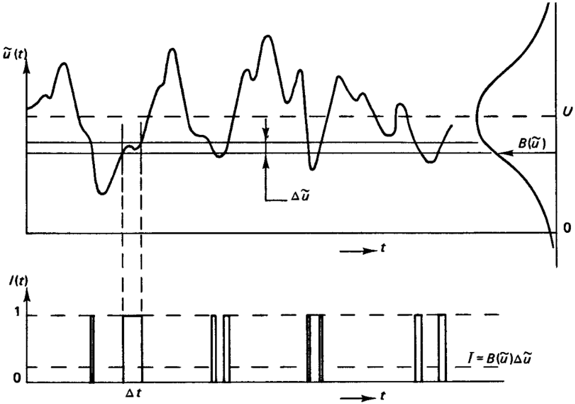

[Tennekes & Lumley, 1972]

Application of central-limit theorem (CLT) [Trotter, 1959],[Lumley, 1976],

$$V \sim N(\mu, \sigma)$$

Method of Gaussianization

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a, b are determined by two calibration data points.

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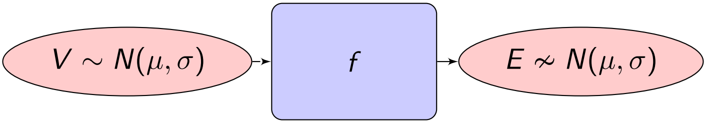
Calibration Procedure Diagram



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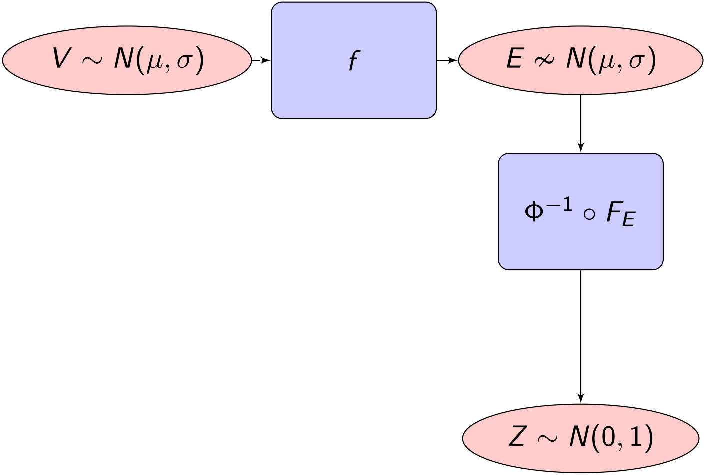

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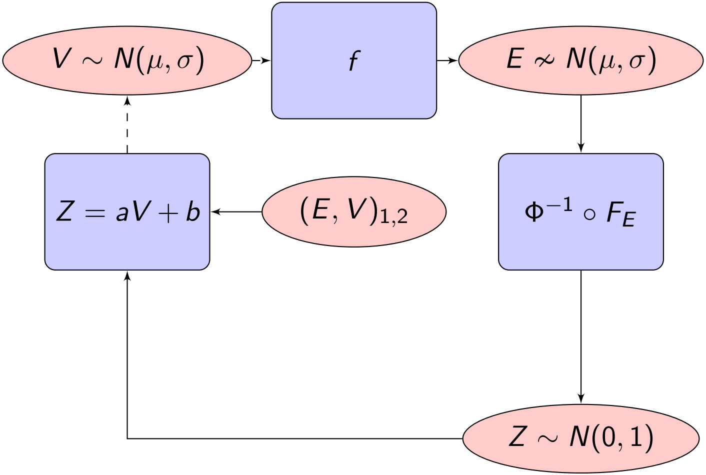

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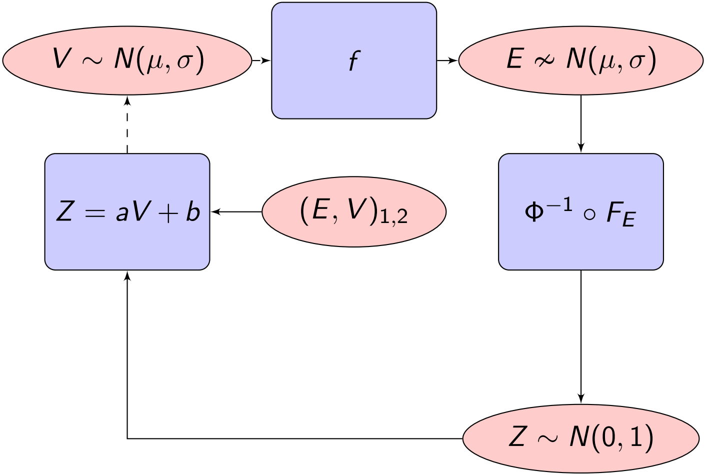
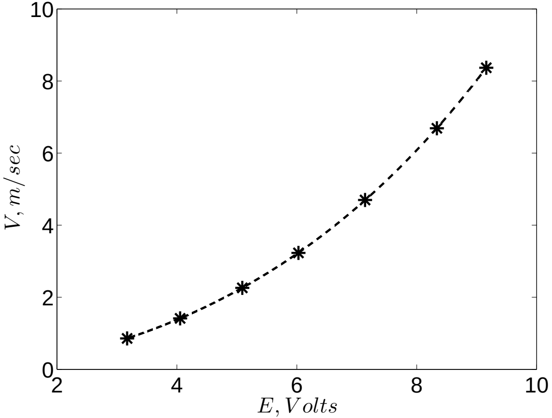


$$V_{est_i} = g_{est}(E_i) = \sum_{k=0}^{4} p_{g_{est},k} E_i^k, \ i = 1, 2, ..., 9$$

Validation

- Tungsten wire of 5 μm diameter and 1 mm length.



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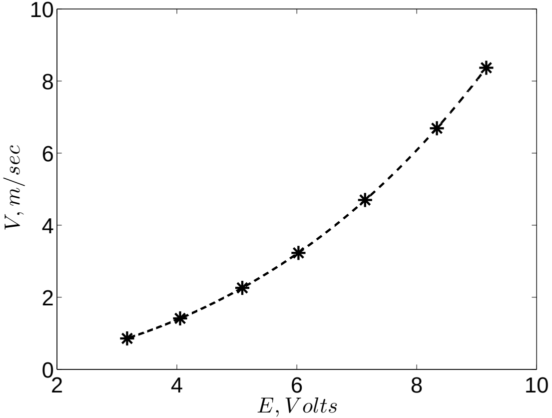

Validation

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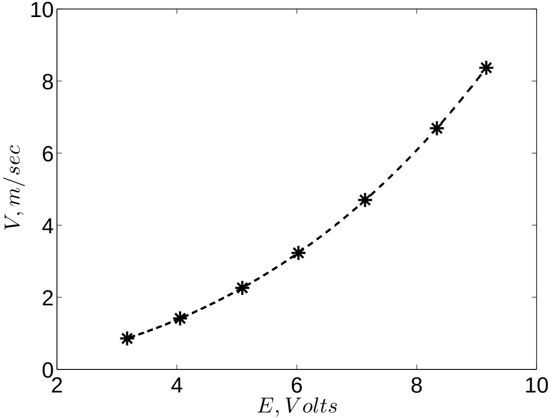
- CTA configuration.
- Calibrated in the free stream of the wind tunnel using a Pitot-static tube in the velocity range of $1 - 10$ m/sec.



Signal Generation

Statistical properties of Gaussian signal: $\gamma_1 = 0$ and $\gamma_2 = 3$ ([Krishnan (2006)]).

mean: $\mu_X = E\left[X(t)\right]$, std: $\sigma_V = [E\left(X - \mu_X\right)^2]^{0.5}$,
skewness: $\gamma_1(X) = E\left[\left(X - \mu_X\right)^3\right]/\sigma^3$, kurtosis: $\gamma_2(X) = E\left[\left(X - \mu_X\right)^4\right]/\sigma^4$.

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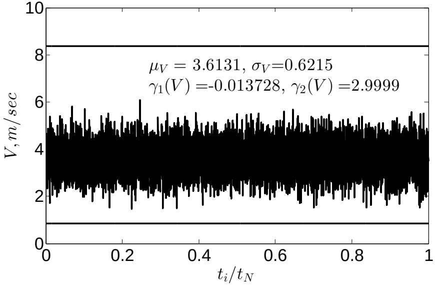

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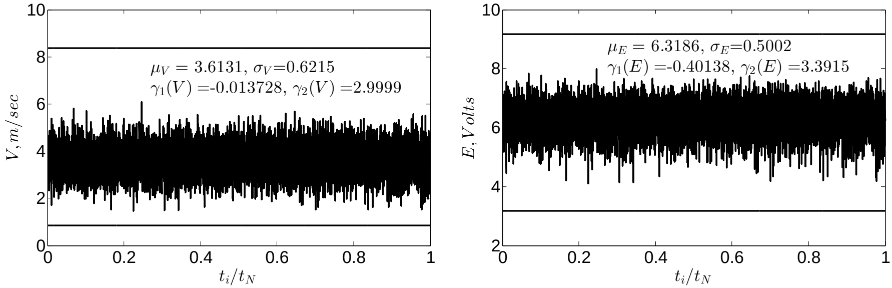

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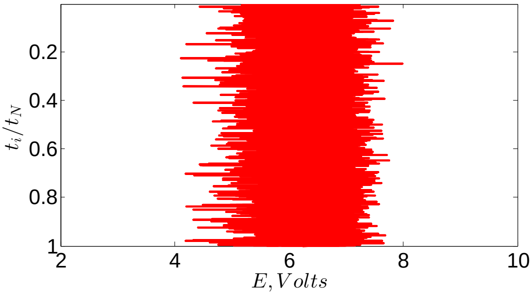

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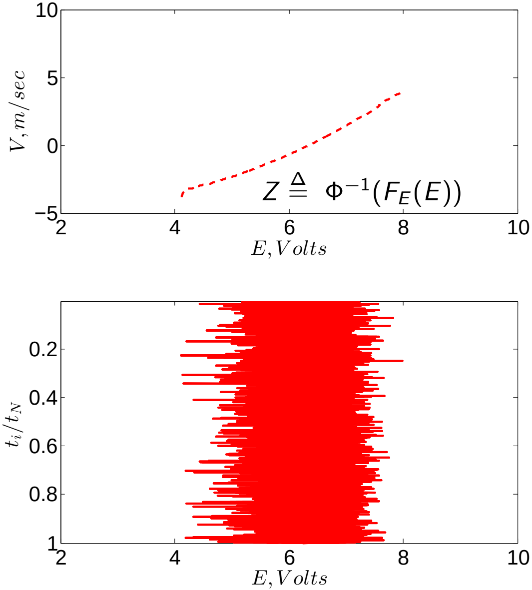

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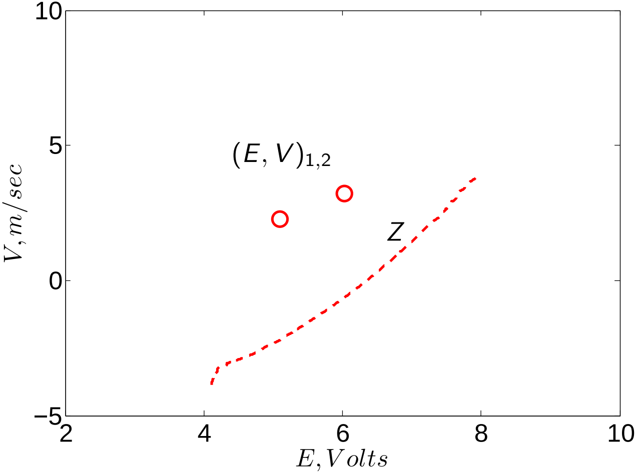

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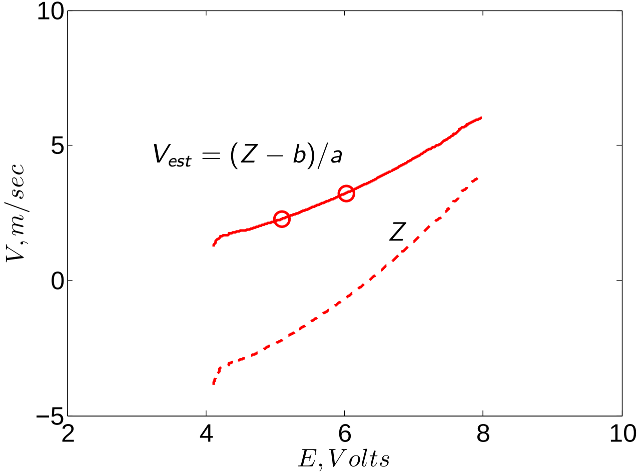

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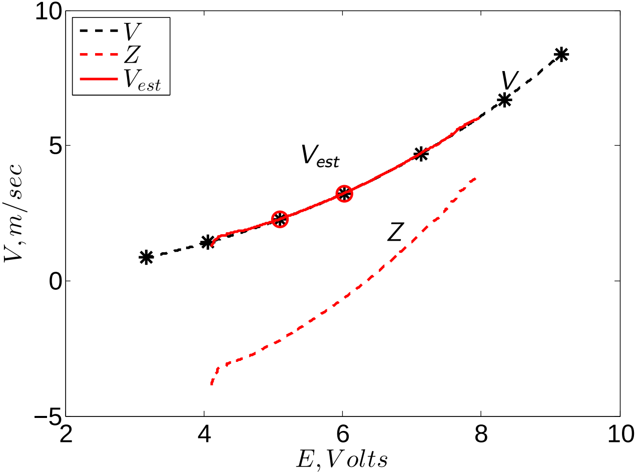

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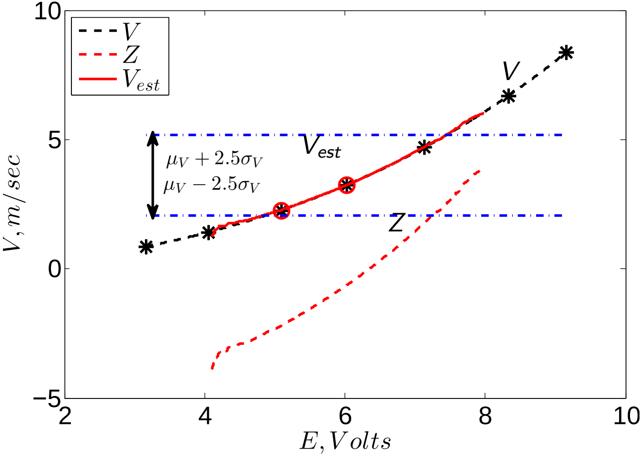

10
5
0
−5
2
4
6
8
10
$V, m/sec$
$E, Volts$
V
Z
V_{est}
$\mu_V + 2.5\sigma_V$
$\mu_V - 2.5\sigma_V$
V
V_{est}
Z

Estimation Error

MSE-discrepancy (error) between $V = g(E)$ and $V_{est} = g_{est}(E)$,

$$MSE = \frac{1}{m - l + 1} \sum_{i=l}^{m} SE_i, \tag{3}$$

SE_i - local square error,

$$SE_i = \left[g\left(E_i\right) - g_{est}\left(E_i\right)\right]^2, \;\; i = l, ..., m. \tag{4}$$

E_l =min$[E(t)]$, E_m =max$[E(t)]$.

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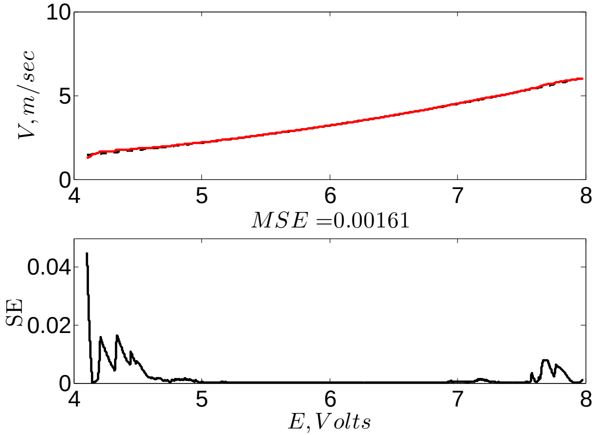

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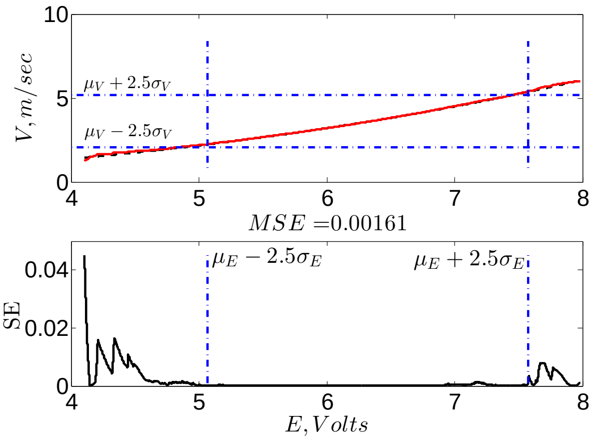

Estimation Performance by Correlation Coefficient

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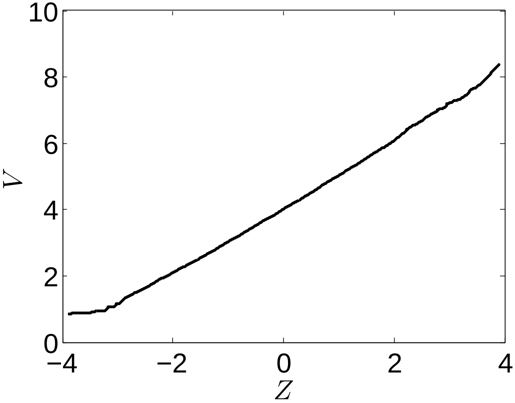
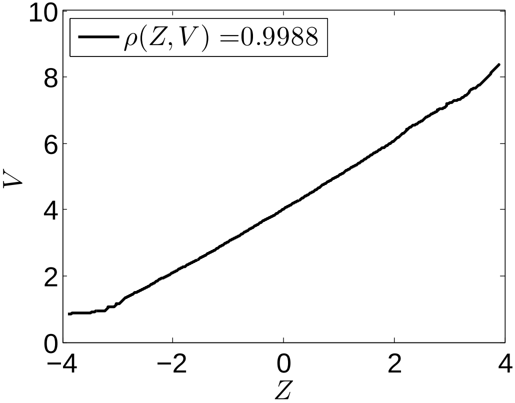


Estimation Performance by Correlation Coefficient

$$\rho = \frac{cov(V,Z)}{\sigma_V \sigma_Z} = \frac{E[(V-\mu_V)(Z-\mu_Z)]}{\sigma_V \sigma_Z}, \ |\rho| \leq 1. \quad (3)$$



Sensitivity of The Method to Gaussian Assumption

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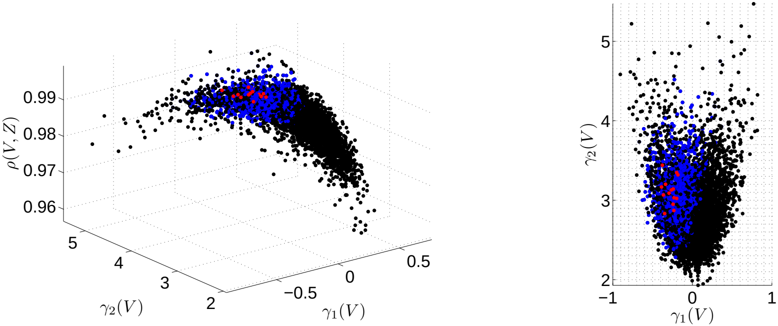
Study the effect of: $\gamma_1(V(t))$, $\gamma_2(V(t))$, $V(t)$.
$V(t)$ is generated 5000 times, 100 data points are used.

Sensitivity of The Method to Gaussian Assumption

$|\rho| - 1 \leq 1 \cdot 10^{-3}$, $1 \cdot 10^{-3} \leq |\rho| - 1 \leq 5 \cdot 10^{-3}$, $|\rho| - 1 > 5 \cdot 10^{-3}$



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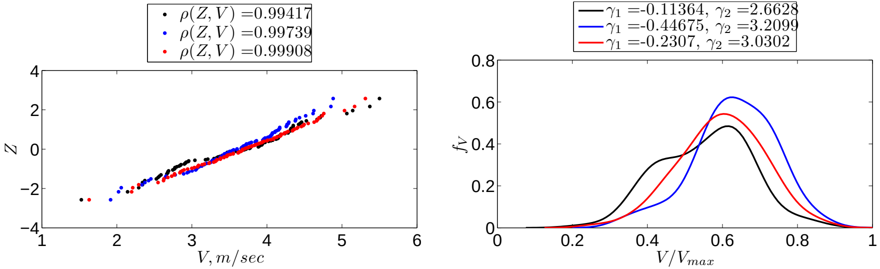

- Method is sensitive to flow velocity distribution.

Sensitivity of The Method to Gaussian Assumption

- Method is sensitive to flow velocity distribution.
- The method can be generalized for other distributions besides Gaussian one,

$$Z = F_V^{-1}(F_E(E))$$

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- Normal velocity distribution can be achieved if the hot-wire sensor is placed inside a turbulent flow regime under certain conditions.
- The method is sensitive to flow velocity distribution.
- The method can be modified to accommodate distributions other than Gaussian and will perform well if the flow distribution is known.

The End

Acknowledgments,
Oleg Kan and Yafim Shulman.

References

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Method Application- Pdf View