Project Report
Transportation of a Slung Load Using Multiple Quadrotors While Maintaining a Constant Altitude Above The Ground

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Abstract

In this project the problem of carrying a single load using multiple quadrotors is being examined. The main goal is to build a control system that will enable us to transport the load safely while maintaining a constant altitude above the ground. The report reviews the design of the simulation environment - the dynamic model of the quadrotors, load and the wires connecting between the load and the quadrotors. The report details the calculation of the trim state and initial conditions for a group of quadrotors carrying a slung load. The trim solution was used to obtain a linear model of the quadrotor and controller design. Simulation results of a controlled single quadrotor and of three or four uncontrolled quadrotors connected to a load system at the trimmed initial condition are presented. The last sections include a review of possible control methods for the full system that was found in the literature and a summary of the work.

1 Introduction

The subject of load transportation using a flock of unmanned aerial vehicles (UAV) has been studied and developed in recent years, [1–4]. We will focus on using quadrotors as our UAVs, however, the theory and methods can be implemented for a large variety of multi-rotor platforms. Carrying a load using multiple small UAVs allows to significantly increase the maximum weight that can be carried using small quadrotors. Moreover, cooperation between multiple agents, although challenging, has many uses beside this project and thus is of great interest in the multi-rotor community.

In this report, modeling of the dynamics of a single quadrotor will be reviewed, followed by a model of a load attached to multiple quadrotors. Dynamic model of the wires will also be presented. Independently, a method to calculate the trim state and the initial conditions while maintaining certain constraints (for an example, safety distance between two quadrotors) will be shown. A controller for a single quadrotor was designed and tested for different reference inputs. The last sections will provide simulation results of the uncontrolled three and four quadrotors connected to a load system. The report will be concluded with a summary of the work performed during the first semester of the project, and a brief introduction to further research.

2 Dynamic model of the quadrotor

The dynamic model of the quadrotor was taken from [5]. It should be noted that, in contrary to [5], the body frame of each quadrotor is set to be in a X configuration and not aligned with the rotors (see Fig. 1). The parameters of the quadrotor are given in Table 1.

The equations of motion of the quadrotor are

$$\begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{Bmatrix} = -\begin{Bmatrix} 0 \\ 0 \\ g \end{Bmatrix} - \frac{1}{m}\begin{bmatrix} A_x & 0 & 0 \\ 0 & A_y & 0 \\ 0 & 0 & A_z \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix} + \frac{D_e^q}{m}\begin{Bmatrix} 0 \\ 0 \\ T \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{Bmatrix} = I_q\begin{Bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{Bmatrix} + \begin{Bmatrix} p \\ q \\ r \end{Bmatrix} \times \left(I_q \begin{Bmatrix} p \\ q \\ r \end{Bmatrix} \right) \tag{2}$$

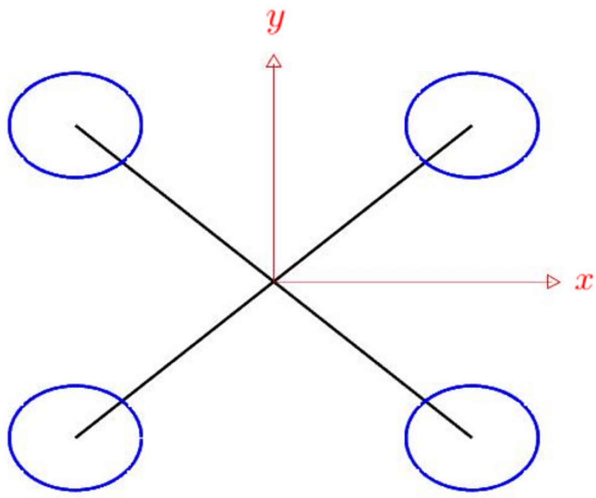

Figure 1: Quadrotor body frame coordinate system

where

$$I_q = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

is the tensor of moments of inertia of the quadrotor. D_e^q is the direction cosine matrix that transfer a body frame vector to inertial frame vector. A_x, A_y, A_z are the drag coefficients linear in the quadrotor velocity. p, q, r are the body angular velocities. T is the thrust generated by the rotors and $\tau_\phi, \tau_\theta, \tau_\psi$ are the quad moments around x, y and z accordingly. m is the load mass.

At this point, the only external forces that act on the quadrotor are the gravity force, drag and the thrust. It is assumed that the thrust and moments generated by the rotors are linear functions of the square of the rotor spin rates $\omega_i, i = 1, 2, 3, 4$ and are expressed as

$$\begin{Bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \\ T \end{Bmatrix} = M_C \begin{Bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{Bmatrix}, \tag{3}$$

here, the matrix M_C stands for "Motor Coefficients" and represented as

$$M_C = \begin{bmatrix} \frac{1}{4kL\cos(45°)} & -\frac{1}{4kL\cos(45°)} & -\frac{1}{4kL\cos(45°)} & \frac{1}{4kL\cos(45°)} \\ -\frac{1}{4kL\cos(45°)} & -\frac{1}{4kL\cos(45°)} & \frac{1}{4kL\cos(45°)} & \frac{1}{4kL\cos(45°)} \\ -\frac{1}{4b} & \frac{1}{4b} & -\frac{1}{4b} & \frac{1}{4b} \\ \frac{1}{4k} & \frac{1}{4k} & \frac{1}{4k} & \frac{1}{4k} \end{bmatrix}$$

where k is the rotor lift constant, b is the rotor drag constant and L is the distance between a rotor and the quadrotor center of gravity (c.g.). Each motor dynamics was approximated as a first order transfer function

$$\frac{\omega}{\omega_{com}}(s) = \frac{1}{1 + \tau s} \tag{4}$$

where τ represents the time constant of the motor. Once the load is added to the model, we insert the forces and moments applied by the cable connected to each quadrotor as external forces and moments.

3 Dynamic model of the load

The load was modeled using the equations of motion, following the same principle as the modeling of the quadrotor, however, without control inputs (following the fact that the load has no propulsion mechanism). The load is affected only by external forces and moments - those that each quadrotor applies and the ones caused by gravity and drag. The load parameters are given in Table 2.

Table 1: Quadrotor parameters

Parameter	Value	Units	Interpretation
g	9.81	$\frac{m}{s^2}$	earth's gravity
m	0.468	kg	quadrotor mass
L	0.225	m	distance between a rotor and the quadrotor center of gravity
k	$2.98 \cdot 10^{-6}$	-	rotor lift constant
b	$1.14 \cdot 10^{-7}$	-	rotor drag constant
I_M	$3.357 \cdot 10^{-5}$	$kg \cdot m^2$	motor moment of inertia
I_x	$4.856 \cdot 10^{-3}$	$kg \cdot m^2$	quadrotor moment of inertia around x
I_y	$4.856 \cdot 10^{-3}$	$kg \cdot m^2$	quadrotor moment of inertia around y
I_z	$8.801 \cdot 10^{-3}$	$kg \cdot m^2$	quadrotor moment of inertia around z
A_x	0.25	$\frac{kg}{s}$	quadrotor drag coefficient in the x direction
A_y	0.25	$\frac{kg}{s}$	quadrotor drag coefficient in the y direction
A_z	0.25	$\frac{kg}{s}$	quadrotor drag coefficient in the z direction
τ	0.1	s	motor dynamics time constant

The load equations are

$$\begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{Bmatrix} = -\begin{Bmatrix} 0 \\ 0 \\ g \end{Bmatrix} - \frac{1}{m_l}\begin{bmatrix} A_{lx} & 0 & 0 \\ 0 & A_{ly} & 0 \\ 0 & 0 & A_{lz} \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix} + \frac{1}{m_l}\sum_{i=1}^{n}\vec{F}_i \tag{5}$$

$$\begin{Bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{Bmatrix} = I_l\begin{Bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{Bmatrix} + \begin{Bmatrix} p \\ q \\ r \end{Bmatrix} \times \left(I_l \begin{Bmatrix} p \\ q \\ r \end{Bmatrix} \right) + \sum_{i=1}^{n}\rho_i \times \vec{F}_i \tag{6}$$

where $\vec{F}_i$ is the force cable i applies on the load (which is, in other words, the force that quadrotor i applies). ρ_i, given and fixed, is the vector originating in the load c.g. and ends at the attachment point between the i'th cable and the load. A_{lx}, A_{ly}, A_{lz} are the drag coefficients of the load and

$$I_l = \begin{bmatrix} I_{lx} & 0 & 0 \\ 0 & I_{ly} & 0 \\ 0 & 0 & I_{lz} \end{bmatrix}$$

is the tensor of moments of inertia of the load.

Table 2: Load and cable parameters

Parameter	Value	Units	Interpretation
m_l	0.25	kg	load mass
I_{lx}	$0.9712 \cdot 10^{-3}$	$kg \cdot m^2$	load moment of inertia around x
I_{ly}	$0.9712 \cdot 10^{-3}$	$kg \cdot m^2$	load moment of inertia around y
I_{lz}	$1.7602 \cdot 10^{-3}$	$kg \cdot m^2$	load moment of inertia around z
A_{lx}	0.3	$\frac{kg}{s}$	load drag coefficient in the x direction
A_{ly}	0.3	$\frac{kg}{s}$	load drag coefficient in the y direction
A_{lz}	0.3	$\frac{kg}{s}$	load drag coefficient in the z direction
ΔX	0.6	m	load length
ΔY	0.6	m	load width
ΔZ	0.1	m	load height
ℓ	4	m	each cable length
K_s	30	$\frac{N}{m}$	cable spring constant
C_d	3	$\frac{kg}{s}$	cable damping constant

4 Wires model

Each wire connecting between each quadrotor to the load was modeled as a spring/damper system as shown in [1]

$$\vec{F_i} = F_{k,i} + F_{c,i}$$

where

$$F_{k,i} = \begin{cases} K_s \Delta\ell_i & \text{if } \Delta\ell_i > 0 \\ 0 & \text{if } \Delta\ell_i \leq 0 \end{cases} \tag{7}$$

and

$$F_{c,i} = \begin{cases} C_s \dot{\ell}_i & \text{if } \dot{\ell}_i > 0 \\ 0 & \text{if } \dot{\ell}_i \leq 0 \text{ or } \Delta\ell_i \leq 0 \end{cases} \tag{8}$$

where $\Delta\ell$ is the extension of the cable with respect to the nominal length ℓ. The values of K_s and C_d displayed in Table 2 were chosen in a way that the wires will be quite stiff, but not exaggerated due to numerical limitations.

5 Trim calculation

Trim means that the sum of forces and sum of moments for each object in the system are zero. This condition interests us for two main reasons - first of all, this is an equilibrium state, which means that when there are no disturbances the system will maintain the specified state. This is the state in which we usually start our simulation at (initial positions, rotor spin rates, Euler angles etc). Secondly, when we will deal with the control problem, we will linearize the system around the trim state, which is the state we aspire to maintain.

We will solve the trim problem for the case of four quadrotors flying in the x direction at a constant speed and under certain constraints (such as safety distance between one another, horizontal load, etc). However, the problem can be easily reformulated to fit a different flying structure, number of vehicles, or desired constraints. It is important to state that more vehicles will require us to set more constraints as we get more degrees of freedom to our system. We will set those constraints in a way that will ensure the quadrotors will not collide.

We start off by counting our equations and parameters - first, we have six equations for each quadrotor equations of motion (24), and six equations originating in the load dynamics (30). However, we have 43 parameters - each quadrotor Euler angles, thrust and moments (28). Each wire contribute three degrees of freedom - two angles that determine its configuration relative to the load and the force it applies (40), and the load Euler angles (43). This means that the problem is under-determined as we have 13 more parameters than equations. In order to close our degrees of freedom we will have to set constraints.

First of all, we will demand that the load will remain horizontal (for an example in the case of sensitive load) which means

$$\phi_{load} = \theta_{load} = \psi_{load} = 0.$$

This assumption provides us with three more equations. We will define two angles for each wire that will determine the wire configuration relative to the load. These Euler-like angles are defined as follows:

- β - will be defined as the angle we rotate the load body frame around the z axis so that the cable will merge with the XOZ plane of the load. We will call that coordinate system "temporary".
- α - will be defined as the angle we rotate the temporary frame around the y axis so that the x axis will merge with the cable direction.

We will assume symmetry around the load XOZ plane (shown in Fig. 2) which means that for a couple of quadrotors, marked with sub-index i and $i+1$ (for $i = 1, 3, 5...m-1$), where $\frac{m}{2}$ is the total number of pairs, we will get the following relations:

$$\beta_i = -\beta_{i+1}, \alpha_i = \alpha_{i+1} \tag{9}$$

and the absolute value of the force of each of their cables will hold:

$$\left|\vec{F_i}\right| = \left|\vec{F}_{i+1}\right| \tag{10}$$

if there is a single front/rear quadrotor (attached in the XOZ plane) we will enforce:

$$\beta_{front} = 0, \beta_{rear} = -180^\circ$$

and their indices will be set as $m+1$ and $m+2$, where the total number of quadrotors in the system is defined to be n. Under those definitions, cable i is represented as the vector

$$\vec{\ell}_i = \begin{Bmatrix} \cos\beta_i \cos\alpha_i \\ \sin\beta_i \cos\alpha_i \\ -\sin\alpha_i \end{Bmatrix} \ell_i \tag{11}$$

and the force applied by the i'th cable on the load will be represented similarly as

$$\vec{F}_i = \begin{Bmatrix} \cos\beta_i \cos\alpha_i \\ \sin\beta_i \cos\alpha_i \\ -\sin\alpha_i \end{Bmatrix} F_i. \tag{12}$$

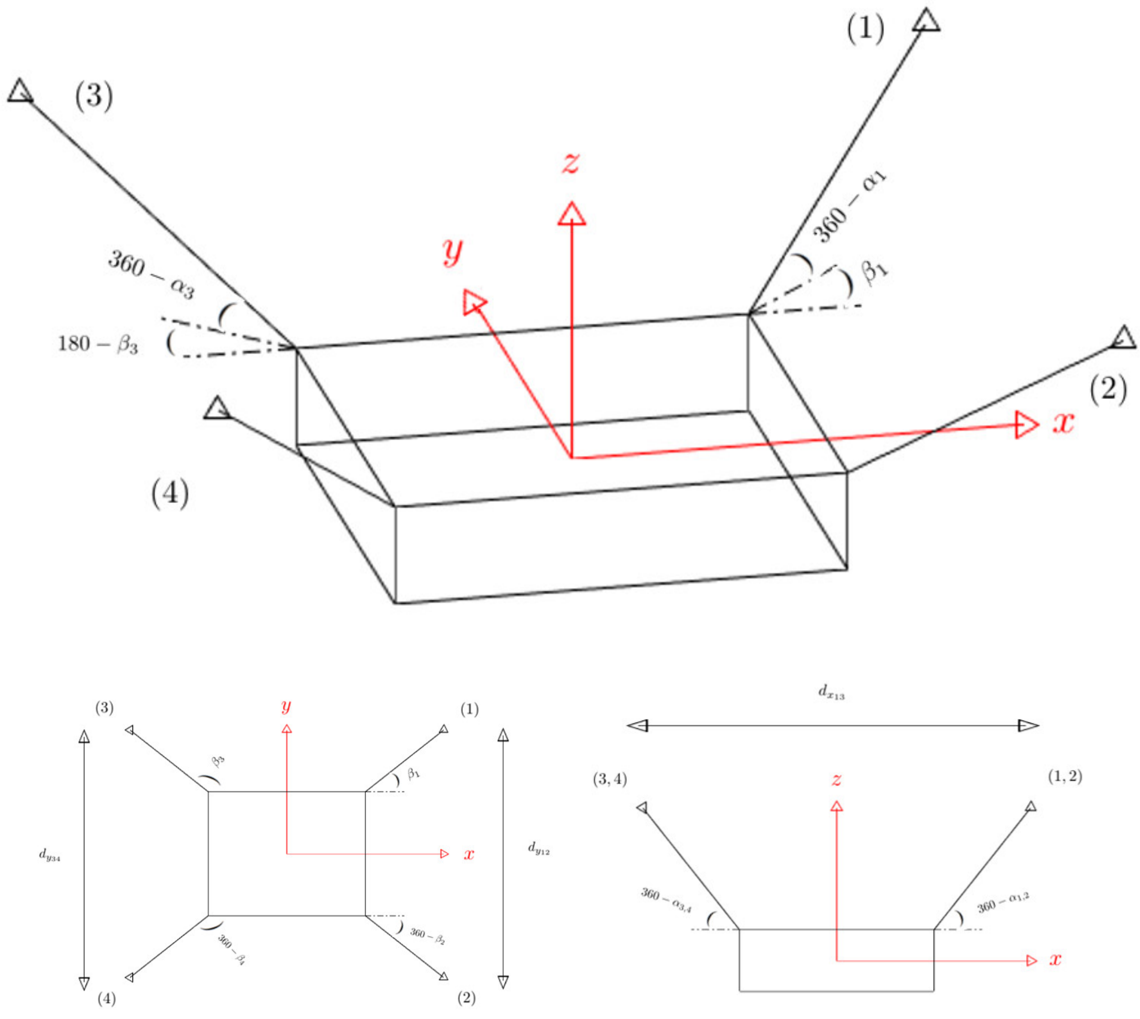

Figure 2: 4 quadrotors connected to a load configuration, definitions of α,β

In our four quadrotors problem, the symmetry assumption (equations 5, 6) will provide us with six more equations, however, it will also eliminate the load moment equations around x and z, and the acceleration equation around y. At this point we are left with 43 parameters and 36 equations. To continue, we will set three constraints on the distances (relating to the distances in the XOY plane of the load) between the quadrotors (for four quadrotors formation, two constraints for three quadrotors, etc.), that will ensure the quadrotors will not collide.

Safety distance between the i'th quadrotor to the j'th quadrotor will be marked as $d_{i,j}$ and it is enough to set three constraints like these. The fourth will be held as a result to the symmetry assumption, this will be showed forward on. (see Fig. 2). Moreover, we will set the yaw angle of all the quadrotors to zero.

With these assumptions the problem has a unique solution - 43 equations and parameters. Once we reduce the immediate equations (the Euler angles that were set to be zero), we are left with 36 equations and parameters. We will show how to solve this system of equations partially numerically and partially analytically. The problem can be decoupled to three sets of equations:

1. Load equations of motion, safety distance constraints and symmetry equations.
2. Each quadrotor acceleration equations
3. Each quadrotor momentum equations

Once solved by order, each set is independent with the later ones. The first set is a highly complex non-linear system of equations that will be solved numerically. The other sets can then be solved analytically. We will now begin with the first set of equations:

Load equations of motion (equations 5, 6)

$$\begin{cases} -A_{lx} + \sum_1^n \cos\beta_i \cos\alpha_i F_i = 0 \\ -m_l g + \sum_1^n \sin\beta_i \cos\alpha_i F_i = 0 \\ \sum_1^n \frac{\Delta Z}{2} \cos\beta_i \cos\alpha_i F_i + \frac{\Delta X}{2} \sin\alpha_i = 0 \end{cases} \tag{13}$$

where ΔX and ΔZ are the load length and height accordingly (this is relevant only for our case displayed in Fig. 2, a more general form is shown at the end of this section - equation 18). As discussed, the rest of the load equations are reduced due to symmetry. The constraint equations

$$\begin{cases} d_{y_{12}} = \ell_1 \sin\beta_1 \cos\alpha_1 + \ell_2 \sin\beta_2 \cos\alpha_2 + \Delta Y \\ d_{y_{34}} = \ell_3 \sin\beta_3 \cos\alpha_3 + \ell_4 \sin\beta_4 \cos\alpha_4 + \Delta Y \\ d_{x_{13}} = \ell_1 \cos\beta_1 \cos\alpha_1 + \ell_3 \cos\beta_3 \cos\alpha_3 + \Delta X \end{cases} \tag{14}$$

where ΔY is the load width. These constraints can be defined differently for any other formation to fit different conditions, number of quadrotors etc. The fourth distance constraint $d_{x_{24}}$ will be held due to the symmetry assumption, one can insert equations 9, 10 and the definition of ℓ_i to the equation of $d_{x_{13}}$ and the equality will be achieved immediately. These six equations, combined with equations 9 and 10 can be solved independently (numerically) to extract α_i, β_i and F_i. After finding these, we can solve the rest of the equations analytically.

The i'th quadrotor equations of acceleration (equation 1)

$$\begin{Bmatrix} -A_x V_x \\ 0 \\ -mg \end{Bmatrix} - \vec{F}_i + D_e^{qi} \begin{Bmatrix} 0 \\ 0 \\ T_i \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{15}$$

it can be shown that

$$\theta_i = \arctan\left(\frac{A_{const,i}}{B_{const,i}}\right)$$
$$\phi_i = \arctan\left(\frac{H_{const,i}}{A_{const,i}} \sin\theta_i\right)$$
$$T_i = -\frac{B}{\cos\theta_i \cos\phi_i}$$

where

$$A_{const,i} = -A_x V_x - \cos\beta_i \cos\alpha_i F_i$$
$$B_{const,i} = -mg + \sin\alpha_i F_i$$
$$H_{const,i} = \sin\beta_i \cos\alpha_i F_i.$$

The i'th quadrotor equations of momentum (equation 2)

$$\vec{M}_{qi} - R_{qi} \times \vec{F}_i = 0 \tag{16}$$

where D_e^b is the transpose of the Direction Cosine Matrix (DCM)

$$D_e^b = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \tag{17}$$

and R_{qi} is the vector connecting between the i'th quadrotor c.g. to the wire attachment point on the quadrotor.

Note that, except for the load momentum equation and the constraints, the equations are general. For completeness, the general momentum equation is

$$\sum_1^n \rho_i \times \vec{F}_i = \vec{0}. \tag{18}$$

From these equations we can extract the desired cable forces and angles (and therefore the full cable vector), and each quadrotor Euler angles and moments. Forward on we can find the desired initial position and rotor spin rates

$$P_{0qi} = P_{0l} + \vec{\ell}_i - D_e^{qi} R_{qi} + D_e^l \rho_i \tag{19}$$

where P_{0qi} and P_{0l} are quad i and the load initial position vector accordingly, assuming the load initial position is given (this equation sets the location in which each quadrotor starts the simulation), and

$$w_{trim,i}^2 = \left\{\vec{M}_{qi} \quad T_i\right\} M_C \tag{20}$$

where $\left\{\vec{M}_{qi} \quad T_i\right\}$ is a 4 components vector in which the first three are the quadrotor moments and the last component is the thrust.

6 Control design

During the project a single quadrotor control system was designed using an independent lead compensator for each ϕ, θ, ψ and a PID (Proportional Integral Derivative)-lead compensator controller for z. The controllers were designed using a linearized model around a static hovering trim condition. The controllers and the system transfer functions are:

ϕ open loop transfer function

$$\frac{\phi}{\tau_\phi}(s) = \frac{2060}{s^2(s+10)}.$$

ϕ controller

$$C_\phi(s) = 0.079\frac{s+0.75}{s+8}.$$

ϕ closed loop

$$\frac{\phi}{\phi_c}(s) = 162.7\frac{(s+0.75)}{(s+12.62)(s+1.58)(s^2+3.8s+6.13)}.$$

Due to the fact that our quadrotor is symmetric, the transfer functions relating to ϕ are identical to those of θ.

ψ open loop transfer function

$$\frac{\psi}{\tau_\psi}(s) = \frac{1136}{s^2(s+10)}.$$

ψ controller

$$C_\psi(s) = 0.143\frac{s+0.75}{s+8}.$$

ψ closed loop

$$\frac{\psi}{\psi_c}(s) = 162.5\frac{(s+0.75)}{(s+12.62)(s+1.58)(s^2+3.8s+6.1)}.$$

z open loop transfer function

$$\frac{z}{T}(s) = \frac{20.9}{s(s+0.534)(s+10)}.$$

z controller

$$C_z(s) = 6.6085 \frac{s+0.55}{s} \frac{s+0.75}{s+7.5}.$$

z closed loop

$$\frac{z}{z_c}(s) = 138 \frac{(s+0.75)(s+0.55)}{s^2(s+10)(s+7.5)(s+0.534)}.$$

Phase margins (PM) and gain margins (GM) for the given controllers are shown in Table 3. Some simulation results are shown in Figs. 3 to 6. As can be seen, the controllers work pretty well for step inputs in ϕ, θ, ψ and z, with decent response time and reasonable overshoot. The controllers also eliminate any steady state errors, relating to the case where there are no disturbances. This controllers might be used in the future as a subsystem for the general multiple quadrotors connected to a load control system.

Table 3: GM and PM for the given controllers

Controller	GM	PM
ϕ	17.3 dB	43.9°
θ	17.3 dB	43.9°
ψ	17.3 dB	43.9°
z	17.9 dB	43.1°

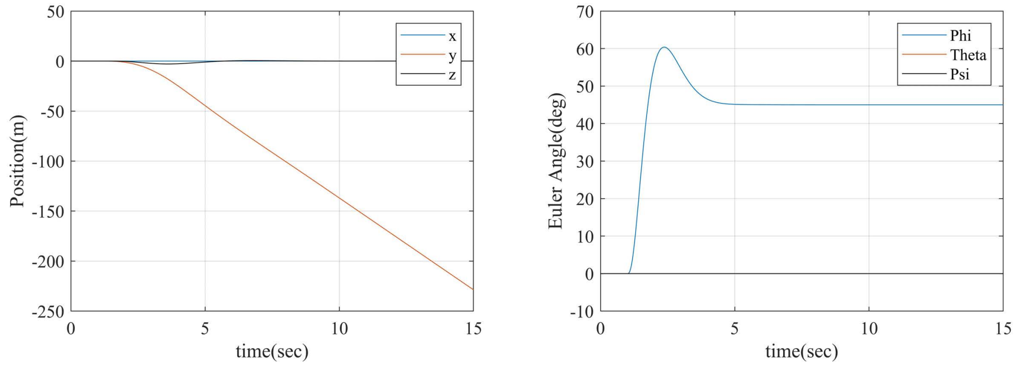

Figure 3: Controlled quadrotor position and attitude for a 45° step input in ϕ at t=1(sec)

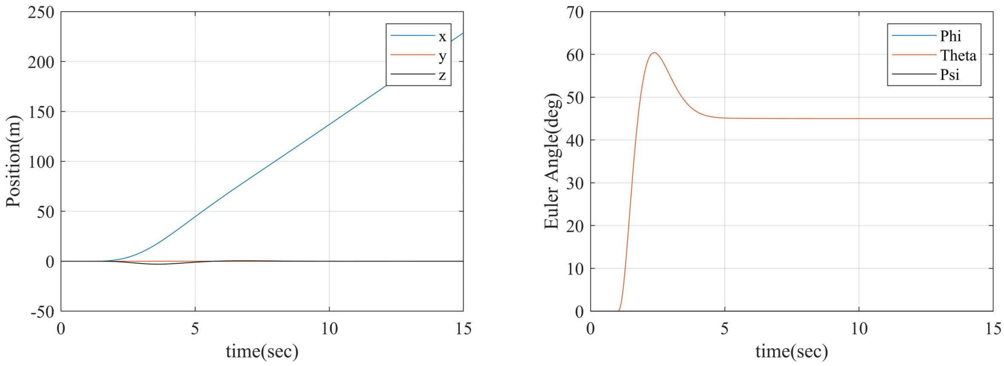

Figure 4: Controlled quadrotor position and attitude for a 45° step input in θ at t=1(sec)

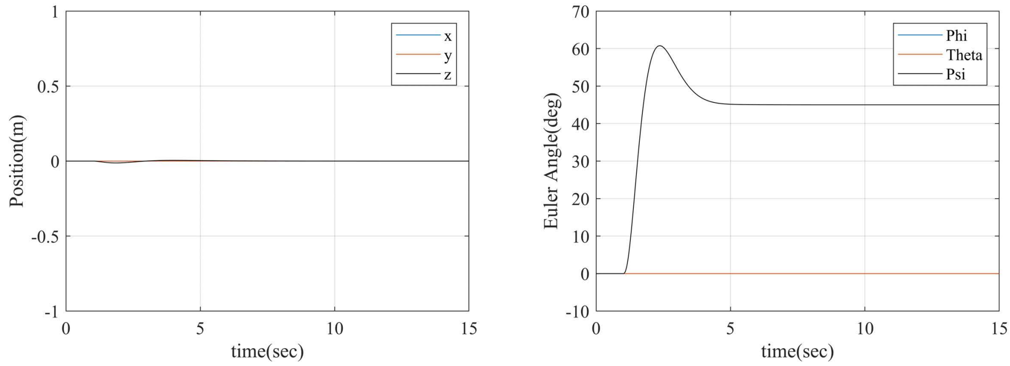

Figure 5: Controlled quadrotor position and attitude for a 45° step input in ψ at t=1(sec)

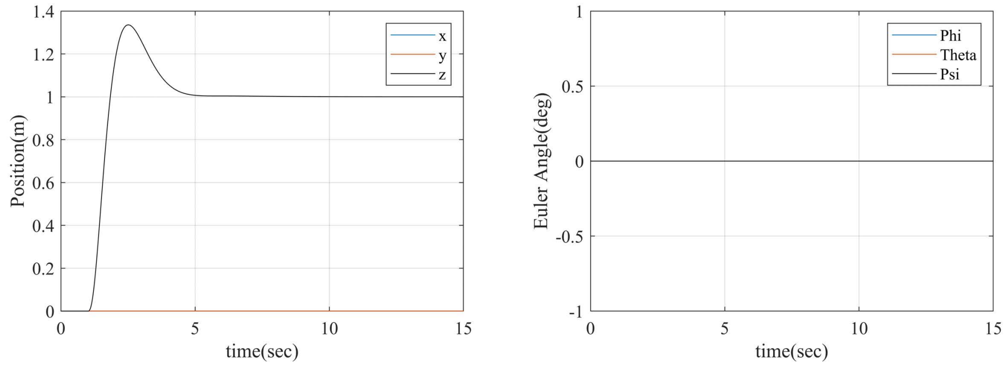

Figure 6: Controlled quadrotor position and attitude for a 1(m) step input in z at t=1(sec)

7 Simulation results

Using the model developed in the previous sections, and the trim calculation described, a simulation for three and four quadrotors configurations connected to a load was modeled. The scenario simulated is the case in which the three or four quadrotors (and the load) are flying at a constant speed in the x direction. The quadrotors and the load in this simulation does not include any kind of control system. The simulation does not consider disturbances of any kind. The relevant system parameters for the simulations are given in Tables 4 and 5.

- As can be seen in Figs. 7 and 9 the three and four quadrotors configurations maintain the trim state for the ideal case - where we start the simulation exactly at the trim conditions and there are no disturbances of any kind.
- The values of C_d and K_s that were chosen yielded that the cable extended lengths were $4.039(m)$ for the 3 quadrotors configuration and $4.020(m)$ for the 4 quadrotors configuration.
- For the case where we have 4 quadrotors and the attachment point of the cable to each quadrotor located at the c.g., the system will drift to the correct trim state even if we start at an offset from the calculated trim point (this result will not be shown in this work). However, if that is not the case, the system is highly unstable and even a small perturbation will cause collision, in order to stabilize the system, control method must be implemented.
- It is important to note that even for the displayed results of the uncontrolled system, after a certain amount of time ($t \approx 100\,(sec)$), the system will eventually diverge from the trim state and collide due to numeric errors.

Table 4: 3 quadrotors configuration parameters

Parameter	Value	Units	Interpretation
P_{0l}	$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$	m	load initial position (c.g.)
ρ_1	$\begin{pmatrix} 0.3 & 0.3 & 0.05 \end{pmatrix}$	m	load attachment points (relative to the load c.g.)
ρ_2	$\begin{pmatrix} 0.3 & -0.3 & 0.05 \end{pmatrix}$	m	load attachment points (relative to the load c.g.)
ρ_3	$\begin{pmatrix} -0.3 & 0 & 0.05 \end{pmatrix}$	m	load attachment points (relative to the load c.g.)
R_{q1}	$\begin{pmatrix} 0 & 0 & -0.02 \end{pmatrix}$	m	attachment point on quadrotor (relative to the quadrotor c.g.)
R_{q2}	$\begin{pmatrix} 0 & 0 & -0.02 \end{pmatrix}$	m	attachment point on quadrotor (relative to the quadrotor c.g.)
R_{q3}	$\begin{pmatrix} 0 & 0 & -0.02 \end{pmatrix}$	m	attachment point on quadrotor (relative to the quadrotor c.g.)
V_{trim}	3.5	$\frac{m}{s}$	trim velocity
$d_{x_{13}}$	1.6	m	safety distance between quadrotors 1 and 3
$d_{y_{12}}$	1.6	m	safety distance between quadrotors 1 and 2

Table 5: 4 quadrotors configuration parameters

Parameter	Value	Units	Interpretation
P_{0l}	$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$	m	load initial position (c.g.)
ρ_1	$\begin{pmatrix} 0.3 & 0.3 & 0.05 \end{pmatrix}$	m	load attachment points (relative to the load c.g.)
ρ_2	$\begin{pmatrix} 0.3 & -0.3 & 0.05 \end{pmatrix}$	m	load attachment points (relative to the load c.g.)
ρ_3	$\begin{pmatrix} -0.3 & 0.3 & 0.05 \end{pmatrix}$	m	load attachment points (relative to the load c.g.)
ρ_4	$\begin{pmatrix} -0.3 & -0.3 & 0.05 \end{pmatrix}$	m	load attachment points (relative to the load c.g.)
R_{q1}	$\begin{pmatrix} 0 & 0 & -0.02 \end{pmatrix}$	m	attachment point on quadrotor (relative to the quadrotor c.g.)
R_{q2}	$\begin{pmatrix} 0 & 0 & -0.02 \end{pmatrix}$	m	attachment point on quadrotor (relative to the quadrotor c.g.)
R_{q3}	$\begin{pmatrix} 0 & 0 & -0.02 \end{pmatrix}$	m	attachment point on quadrotor (relative to the quadrotor c.g.)
R_{q4}	$\begin{pmatrix} 0 & 0 & -0.02 \end{pmatrix}$	m	attachment point on quadrotor (relative to the quadrotor c.g.)
V_{trim}	4	$\frac{m}{s}$	trim velocity
$d_{x_{13}}$	1.6	m	safety distance between quadrotors 1 and 3
$d_{y_{12}}$	1.6	m	safety distance between quadrotors 1 and 2
$d_{y_{34}}$	1.6	m	safety distance between quadrotors 3 and 4

8 Conclusion

As shown in this project, the problem of controlling the multi agent system is not an easy task. It is a highly complex nonlinear MIMO system and a lot of constraints and conditions must be met. During this project we covered the following subjects:

- A way to design the simulation environment for the n quadrotors connected to a load system.
- The procedure for calculating the trim state and initial conditions.
- Controller for a single quadrotor was designed.
- Simulation results for the controlled single quadrotor and for the uncontrolled 3 and 4 quadrotors connected to a load configurations were shown.

Beyond the scope of this project, there has been quite a few control methods that were applied on similar problems in the literature. In [3] a geometric controller was designed to make sure the load carried by a flock of quadrotors follows a certain trajectory. In [4] a LQR (Linear Quadratic Regulator)-PID method was used in order to achieve stability of a similar system while maintaining a certain formation of the flock and following a given trajectory. In [2] a Sliding Mode Control (SMC) method was implemented to ensure stability of a similar system. In further work we will examine and attempt to use the LQR method and other control methods. The goal would be to ensure the stability of the system and to keep the load at a constant altitude above the ground, while maintaining the constraints of the formation and rejecting disturbances.

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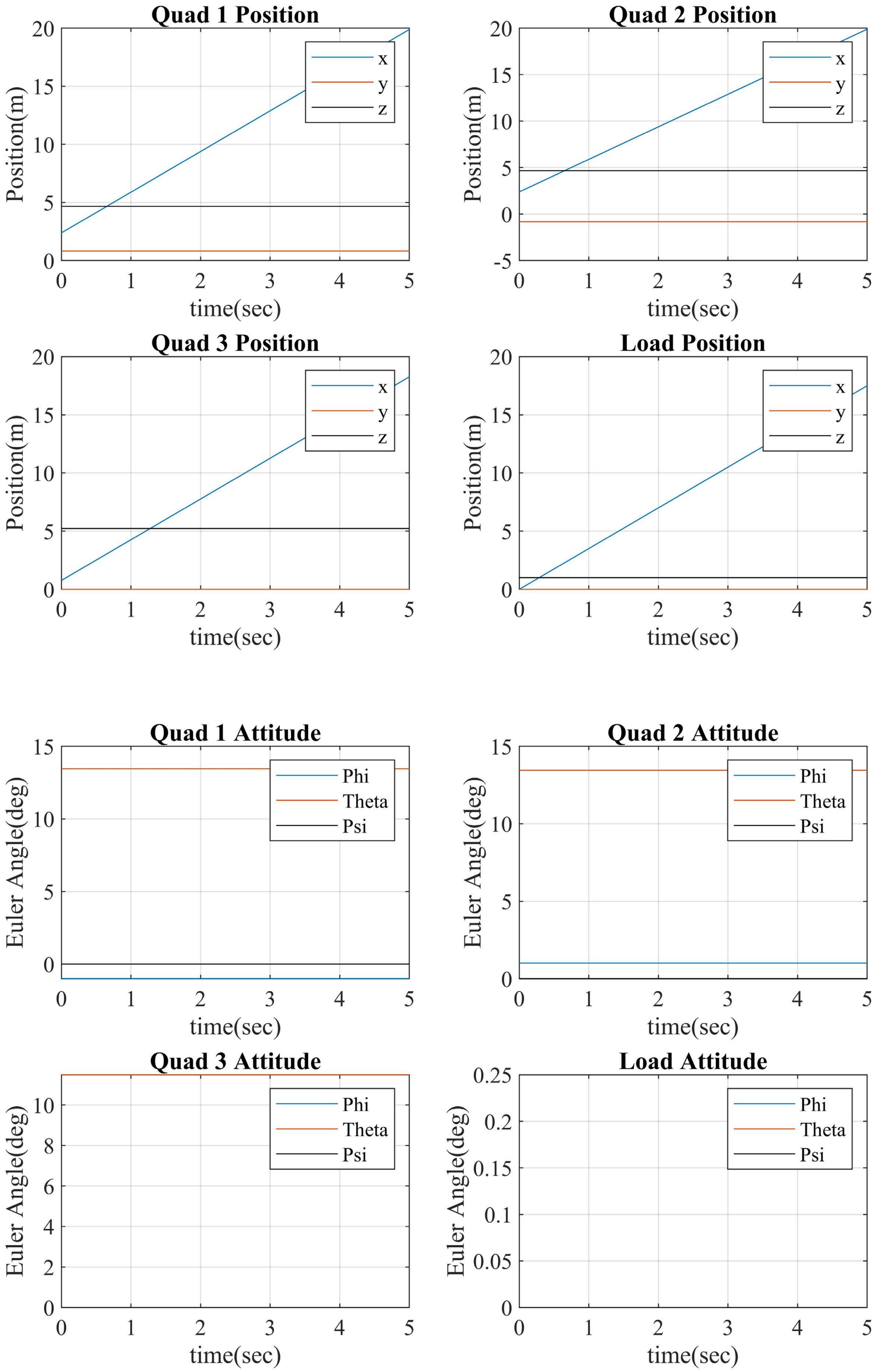

Figure 7: 3 quadrotors configuration position and attitude simulation

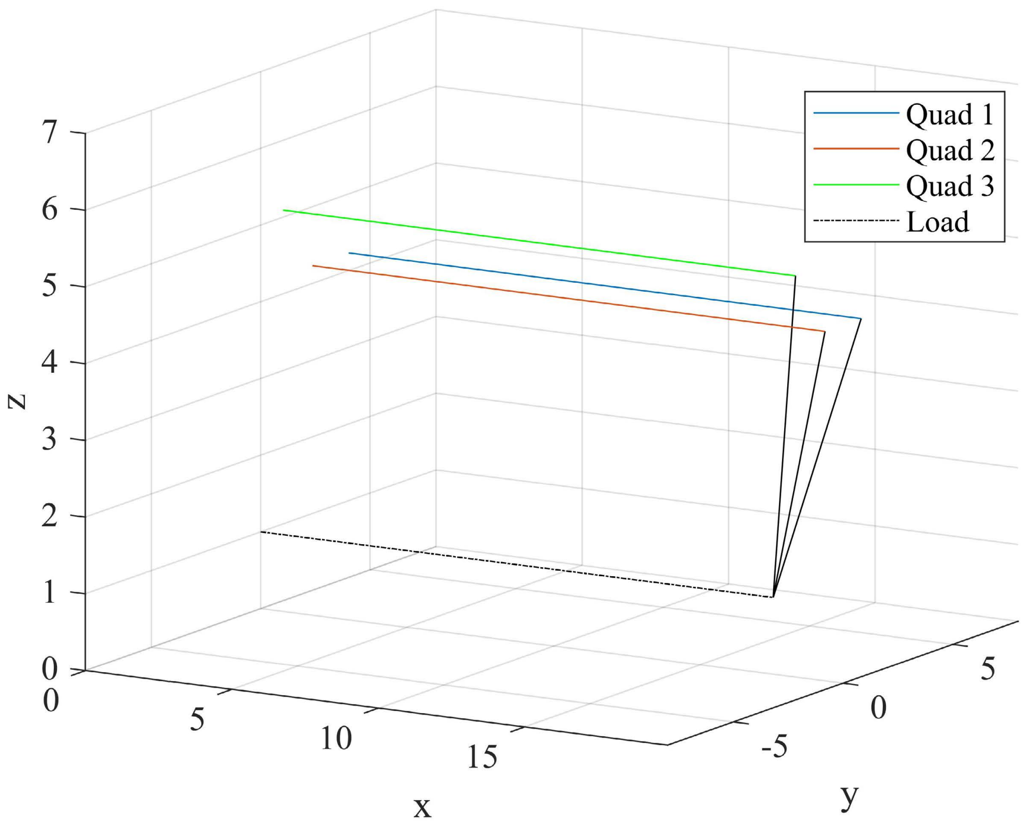

Figure 8: 3 quadrotors configuration 3D graph

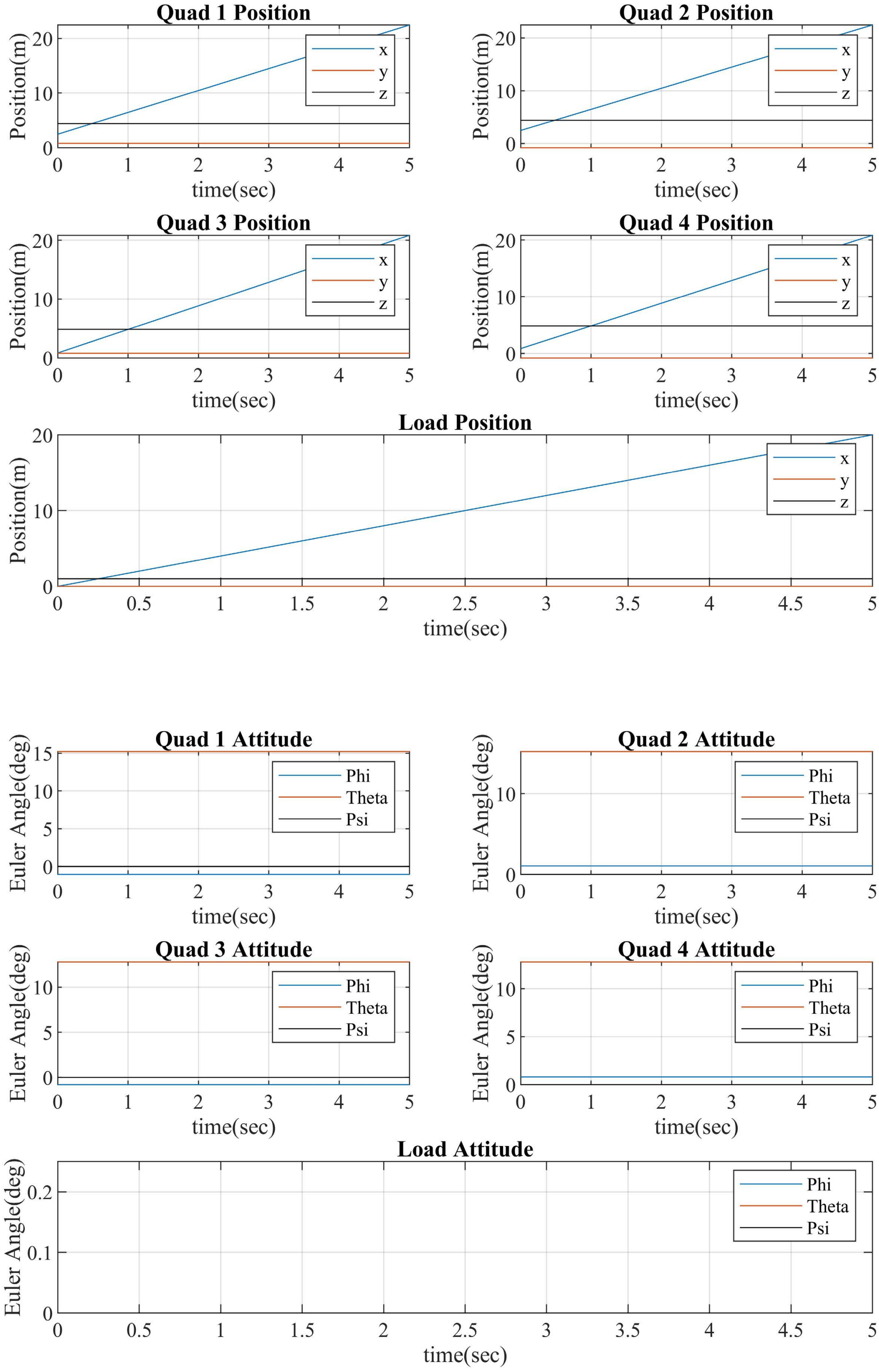

Figure 9: 4 quadrotors configuration position and attitude simulation

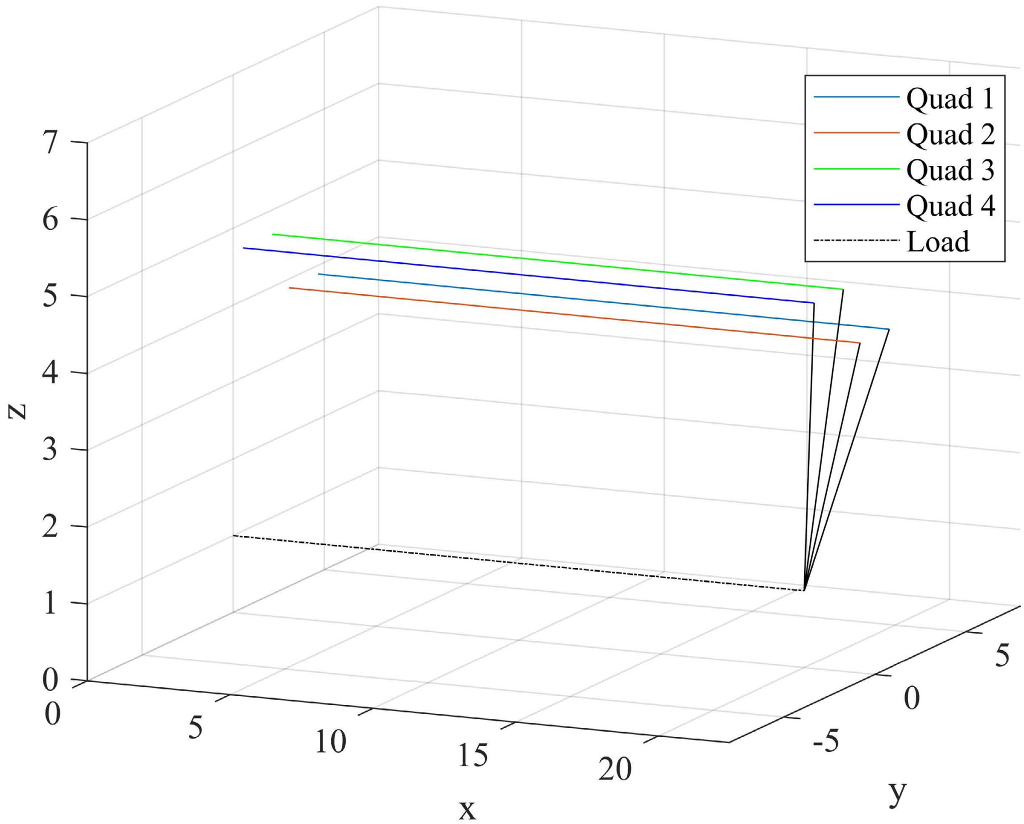

Figure 10: 4 quadrotors configuration 3D graph