Liquid Fuel Behavior in a Spacecraft Fuel Tank During Acceleration/Deceleration

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Abstract

This research studies the accelerating and decelerating behavior of liquid fuel stored in the fuel tank of a spacecraft in the absence of gravity. When the tank is not completely filled with fuel and part of its volume is fuel vapor, a sudden change in the spacecraft motion will cause liquid motion inside the tank. This will compress and/or expand the vapor, assumed to be an ideal gas, altering the vapor properties according to Boyle's Law. Two processes of liquid motion are taken into consideration in this research: isothermal and adiabatic processes. Taking a cylindrical tank moving along its axis, both result in a periodic motion of the liquid slug between two bases of the fuel tank.

1 Introduction

Most of the satellites and spacecrafts are powered by propulsion systems filled with liquid fuels. One of the frequently encountered problems during connection maneuver, for example between spacecrafts and space station, is that the sudden change in the spacecraft motion, acceleration or deceleration, will lead to a change in the liquid fuel motion inside a partially filled fuel tank. These induced motions of liquid will again affect the motions of fuel tank, and thus hinder the smoothness of a desired connection maneuver.

Previous researches have extensively investigated into the sloshing dynamics of liquid fuel in a partially filled tank under micro-gravity situation, both theoretically and experimentally [**1**]. Snyder [**2**] studied the sloshing of cryogens using a surface wave model and its modal response to the tank motion. Agrawal [**3**] studied liquid subjected to centrifugal force in a spinning spacecraft using a boundary layer model. Sloshing induced by the acceleration/braking of road vehicles are also studied by numerically solving potential flow equation and free surface equation [**4**]. The present study focuses on the interaction between liquid fuel and the remaining vapor volume in such scenarios, and the major mechanism affecting liquid motion is the thermodynamic properties of vapors. Equations of states for ideal gas fuel vapor are coupled with equations of motion for the incompressible liquid slug to calculate its time-dependent behaviors and cycle frequencies.

Consider a liquid volume originally moves together with the partly filled tank, when the tank suddenly stops or starts up, the subsequent motion of liquid fuel is mainly determined by its interaction with the fuel vapor volume in the remaining part of the fuel tank, and also affected by the tank walls. To investigate the motion of liquid, it is essential to investigate the time-dependent thermodynamics properties of vapor volume during acceleration/deceleration. In order to compare the effects of vapor properties on the liquid slug motions, isothermal and adiabatic processes are both examined. And several physical situations are discussed separately.

In the presented research, both the liquid and vapor are treated as incompressible fluids and the vapor is considered as ideal gas. And since the studied objects are in a micro-gravity environment on high orbits, all the body forces are considered negligible compared to vapor pressures on liquid slug.

Notations:

- U_0 - constant tank velocity as defined below in each case
- $U(t)$ - tank velocity as a function of time
- $u(t)$ - liquid volume absolute frame velocity as a function of time
- $a(t)$ - liquid volume absolute frame acceleration as a function of time , $a(t \leq 0) = 0$
- P_0 - initial vapor pressure in the tank
- T_0 - initial vapor temperature in the tank
- d_0 - initial length of vapor
- A_0 - cross-sectional area of the tank.
  When viscous effect is negligible, the cross-sectional area of liquid flow is the same as that of the tank A_0.
- L - total length of the tank
- $d(t)$ - time-dependent length of vapor volume
- $P(t)$ - time-dependent vapor volume
- $T(t)$ - time-dependent vapor temperature
- ρ_l - liquid fuel density, treated as a constant in this incompressible flow study

2 Analytical Model

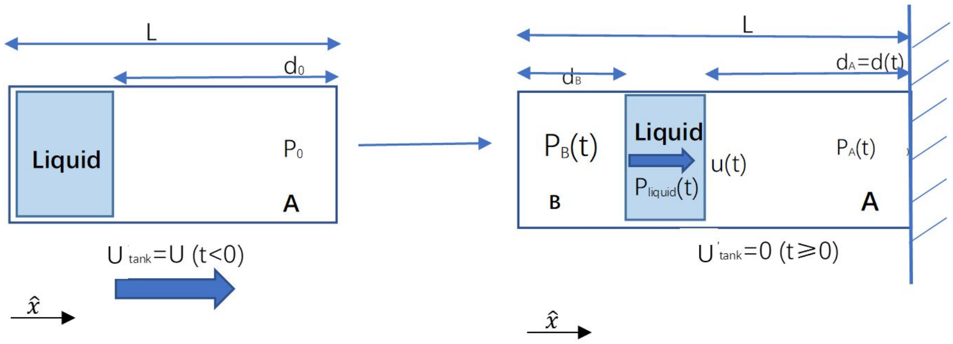


Figure 1: Schematic description of the problem. On the left is the original state where liquid and vapor move together in the tank; on the right liquid slug decelerates towards the other side of the tank when the tank suddenly stops.

2.1 Governing Equations

The governing equations for an one-dimensional incompressible liquid flow can be written as (in the dimensional form)
continuity

$$\frac{\partial u}{\partial x} = 0 \tag{1}$$

which indicates that the incompressible liquid slug moves together as a whole.
momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho_l}\frac{\partial P_{liquid}}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2} \tag{2}$$

Assuming that the viscous effect is negligible (as will be shown later in this report), the non-linear term drops, equation (2) is then reduced to:

$$a(t) = \frac{\partial u}{\partial t} = -\frac{1}{\rho_l}\frac{\partial P_{liquid}}{\partial x} \tag{3}$$

which becomes the equation of motion describing the acceleration $a(t)$ of the liquid slug.

There exists a relation connecting vapor length in region A, $d(t)$, with the velocity and acceleration of the liquid slug (positive direction defined as in figure 1):

$$u(t) = -\frac{\partial d(t)}{\partial t} \longrightarrow a(t) = -\frac{\partial^2 d(t)}{\partial t^2} \tag{4}$$

Equation (3) shows that $\frac{\partial P_{liquid}}{\partial x}$ has no dependency on the axial location x, the pressure distribution within the liquid slug must be linear and decided by the pressure difference on its two sides:

$$\frac{\partial P_{liquid}v(t)}{\partial x} = \frac{P_A(t) - P_B(t)}{L - d_0} \tag{5}$$

The time-dependent pressures P_A and P_B of region A and B (as shown in figure 1) can be derived from the Boyle's law as a function of their volumes. And since both volume cross-sectional areas remain constant A_0 with negligible boundary layer thickness, their pressures P_A and P_B can be directly expressed as functions of region A vapor length $d(t)$.

Pressure $P_A(t)$ can be found applying Boyle's Law on the vapor mass that already exists in region A. It is assumed that no evaporation occurs throughout the entire first cycle of liquid slug traveling back to its original location. Later calculation results will justify this assumption. Comparing it with its original state (γ being the heat capacity ratio of fuel vapor),
Isothermal

$$P_0 \cdot A_0 d_0 = P_A(t) \cdot A_0 d(t) \tag{6}$$

Adiabatic

$$P_0 \cdot A_0^\gamma d_0^\gamma = P_A(t) \cdot A_0^\gamma d(t)^\gamma \tag{7}$$

$P_B(t)$ is discussed according to two cases:

1. **Case 1**

   In the first case, the liquid slug starts moving from one base of the tank towards to other base, and hence no vapor mass exists in region B before the tank suddenly changes its motion, as shown in figure 1. It is assumed that no evaporation occurs throughout the entire first cycle of liquid slug traveling back to its original location. Later calculation results will justify this assumption. Therefore, region B can be constantly treated as a vacuum volume and $P_B(t) = 0$ during the first cycle. Equation (3) finally becomes a second order ODE for $d(t)$:
   Isothermal

$$\frac{\partial^2 d}{\partial t^2} = \frac{1}{\rho_l} \frac{P_0(\frac{d_0}{d})}{L - d_0} \tag{8}$$

   Adiabatic

$$\frac{\partial^2 d}{\partial t^2} = \frac{1}{\rho_l} \frac{P_0(\frac{d_0}{d})^\gamma}{L - d_0} \tag{9}$$

   with initial conditions $d(0) = d_0$ and $d'(0)$ depending on specific cases.

2. **Case 2**

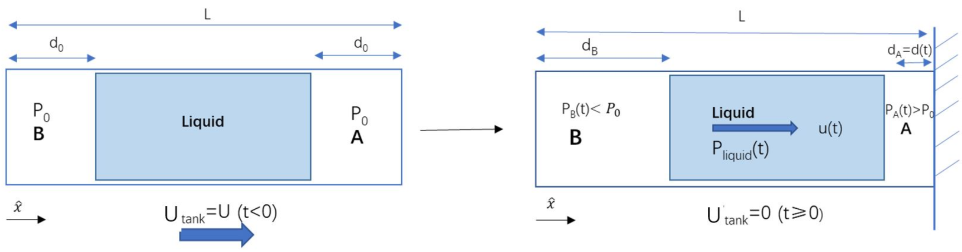


Figure 2: Schematic description of the second case of the problem. The liquid is now originally located in the middle of the tank in some way. Equal amount of vapors exist in region A and B, with the same length d_0.

   Figure (2) shows that the original vapor mass in region B is greater than zero. With negligible evaporation of liquid fuel, pressure $P_B(t)$ can be written from the Boyle's Law comparing to its own original state:
   Isothermal

$$P_0 \cdot A_0 d_0 = P_B(t) \cdot A_0 d_B(t) \tag{10}$$

Adiabatic

$$P_0 \cdot A_0^\gamma d_0^\gamma = P_B(t) \cdot A_0^\gamma d_B(t)^\gamma \tag{11}$$

where $d_B(t) = 2d_0 - d(t)$.

Equation (3) becomes a second order ODE for $d(t)$:
Isothermal

$$\frac{\partial^2 d}{\partial t^2} = \frac{1}{\rho_l} \frac{P_0(\frac{d_0}{d}) - P_0(\frac{d_0}{2d_0 - d})}{L - d_0} \tag{12}$$

Adiabatic

$$\frac{\partial^2 d}{\partial t^2} = \frac{1}{\rho_l} \frac{P_0(\frac{d_0}{d})^\gamma - P_0(\frac{d_0}{2d_0 - d})^\gamma}{L - d_0} \tag{13}$$

with initial conditions $d(0) = d_0$ and $d'(0)$ depending on specific cases.

Assuming the adiabatic processes are isentropic, the temperature change of vapor volume in region A can be calculated according to the relation:

$$\frac{T_A(t)}{T_0} = \left(\frac{V_0}{V_A(t)}\right)^{\gamma - 1} = \left(\frac{d_0}{d(t)}\right)^{\gamma - 1} \tag{14}$$

For the isothermal process, $T_A(t) = T_0$.

The solution to $d(t)$ will indicate the time required to reach the other base, maximal velocity throughout liquid slug's motion during the first cycle and maximal temperature the vapors can reach. The numerical results will help decide if all the aforementioned assumptions are valid or not by comparing them to some scales.

2.2 Dimensional Scales

The non-dimensional variables are defined as:

$$\tilde{x} = \frac{x}{L}, \qquad \tilde{t} = \frac{t}{L/U_0}, \qquad \tilde{u} = \frac{u}{U_0}, \qquad \tilde{P} = \frac{P}{P_0}, \qquad \tilde{T} = \frac{T}{T_0}, \qquad \tilde{d_A} = \frac{d}{L}, \qquad \tilde{d_B} = \frac{2d_0 - d}{L}$$

The dimensionless governing equations become:
case 1

$$\frac{\partial^2 \tilde{d}}{\partial t^2} = \frac{P_0}{\rho_l U_0^2} \cdot \frac{\tilde{d_0}^\gamma}{1 - \tilde{d_0}} \cdot \frac{1}{\tilde{d}^\gamma} \tag{15}$$

case 2

$$\frac{\partial^2 \tilde{d}}{\partial t^2} = \frac{2P_0 \tilde{d_0}}{\rho_l U^2 \left(1 - 2\tilde{d_0}\right)} \frac{\tilde{d_0} - \tilde{d}}{\tilde{d}\left(2\tilde{d_0} - \tilde{d}\right)} \tag{16}$$

Note that here isothermal case is represented by when $\gamma = 1$. And dimensionless temperature:

$$\tilde{T}_{\text{isentropic}} = \left(\frac{\tilde{d_0}}{\tilde{d}}\right)^{\gamma - 1} \tag{17}$$

3 Results

Results of characteristics of liquid slug motion during the first cycle are discussed according to the following four physically possible situations. All the numerical results are obtained using MATLAB solver ODE45. Liquid slug cycling motion frequencies with different liquid portions existing in the tank are compared.

1. **Situation 1**

   Consider a tank with $D = 0.5$m diameter and $L = 2$m length, and gasoline liquid with density $\rho = 780\text{kg/m}^3$, kinematic viscosity $\nu = 6 \cdot 10^{-7}\text{m}^2/\text{s}$, thermal diffusivity $\alpha = 0.462 \cdot 10^{-6} m^2/s$ and initial temperature at $T_0 = 288$K.

   Consider the case where gasoline liquid fuel fills 25% of the total tank volume and initially located on one side of the tank. The tank moves at $U_{tank} = U_0 = 2$m/s for $t < 0$, then it stops suddenly with $U_{tank} = 0$m/s for $t \geq 0$, as shown below.

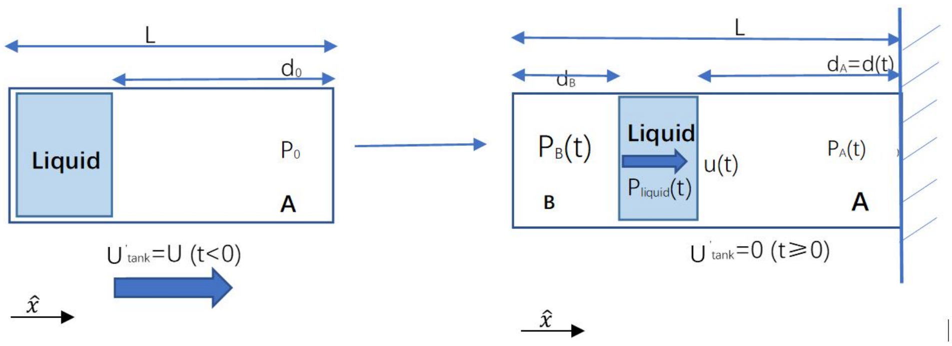


Figure 3: Schematic description of the situation.

With initially no vapor mass in region B, ODE (15) applies in this situation with initial conditions $\tilde{d}(0) = \tilde{d}_0$ and $\tilde{d}'(0) = -\tilde{u}(0) = -1$. Parameters used for calculation are listed below. The

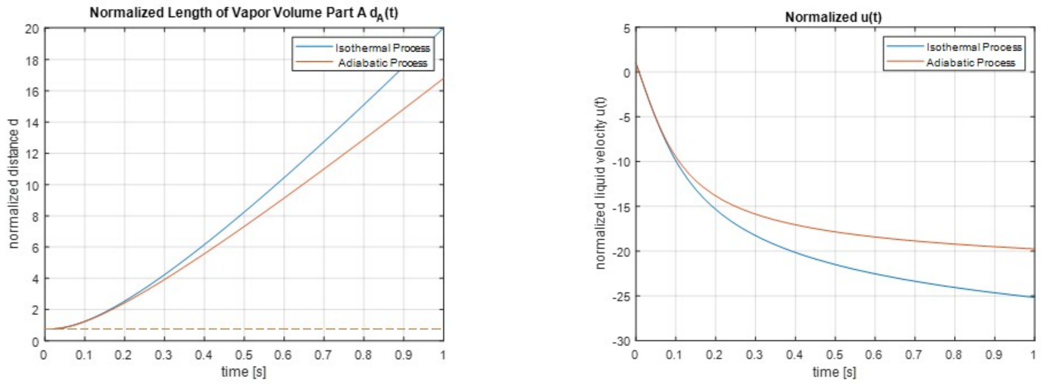
P_0	ρ_l	U_0	$\tilde{d}_0$	γ
$10^5 Pa$	780 kg/m^3	2 m/s	0.75	1.3

Table 1: Calculation Parameters

time-dependency of length $d(t)$ and liquid slug velocity $u(t)$ during the first cycle is shown in figure (4a) and (4b). Result presented in figure (4) is not reasonable since $\tilde{d}_A$(t) should always be less than 1. The explanation is that the initial velocity of the liquid $u(t = 0) = U_0 = 2$m/s is too low and the pressure in region A P_A is large enough, compared to P_B, to decelerate the liquid block immediately to zero, which results in both the tank and the liquid staying stationary.

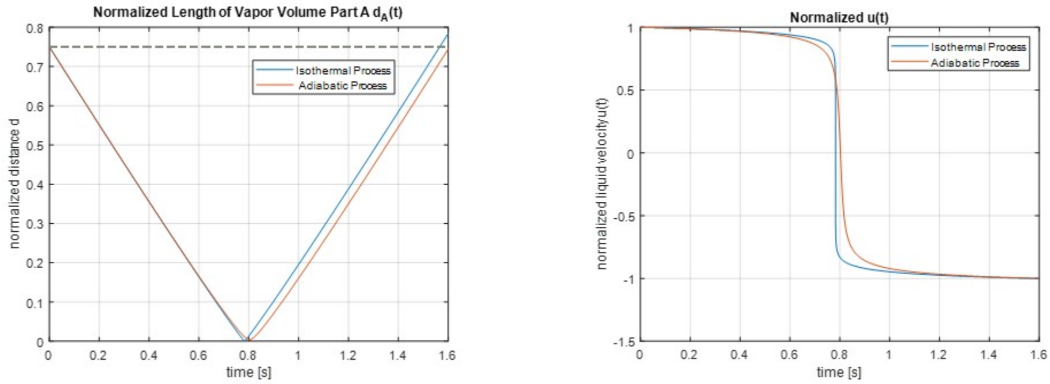
In this case, since the vapor volume experiences almost no change at all time, its temperature can also be seen as constant throughout the process, $T_A(t) = T_0$.

If the initial velocity is larger, $u(t = 0) = U_0 = 100$m/s, then the pressure P_A will not be large enough to decelerate it immediately. In this case, while decelerating, the liquid will move almost to the other side of the tank until u(t)=0 and then moves backwards, as shown in figure (5).



(a) Normalized d(t) (b) Normalized u(t)

Figure 4: Situation 1 with initial velocity $U_0 = 2$ m/s



(a) Normalized d(t) (b) Normalized u(t)

Figure 5: Situation 1 with initial velocity $U_0 = 100$ m/s

As seen from figure (5a), the required time to move to the other side of the tank is approximately $\Delta t = 0.8s$ and the distance covered is almost $\Delta d = 0.75L = 1.5$m. In this case, the Reynolds number $Re_D = \frac{U_0 D}{\nu} = 8.33 \cdot 10^7$ and the liquid flow is turbulent. The approximate entry length of liquid slug into the tank $L_e = 1.359 Re_D^{1/4} \approx 129.8\text{m} \gg L = 2\text{m}$ is much longer than the tank total length. So the flow velocity profile is far from developed. The distance covered during deceleration Δd only 1.16% of total entry length, so the spatial change in velocity profile $u(r)$ along the tank is negligible and can be considered uniform, i.e. $u(r) = u(t)$ for $0 \leq r \leq R$.

Furthermore, in order to develop a steady state parabolic profile, a certain amount of time is required. The diffusive time scale is defined as $\tau_{diff} = \frac{R^2}{\nu} = 104167s$, which is also much longer than the deceleration time $\Delta t = 0.8s$ calculated above. Therefore, the flow does not have enough time or space to experience significant change in its axial velocity profile, and the viscous effect is thus negligible in this case.

Temperature change in the isentropic process is calculated from equation (17) and is presented in figure (6). The vapor temperature sees a sharp increase when the liquid slug approaches the other base of the tank, and then drops back to T_0 as it moves back. The peak temperature reaches almost $5T_0 \approx 1440K$, while the boiling temperature of liquid gasoline is about $T_{boiling} = 358K$. The time duration of vapor temperature that is above $T_{boiling}$ is $t = 0.4s$ to $t = 1.2s$.

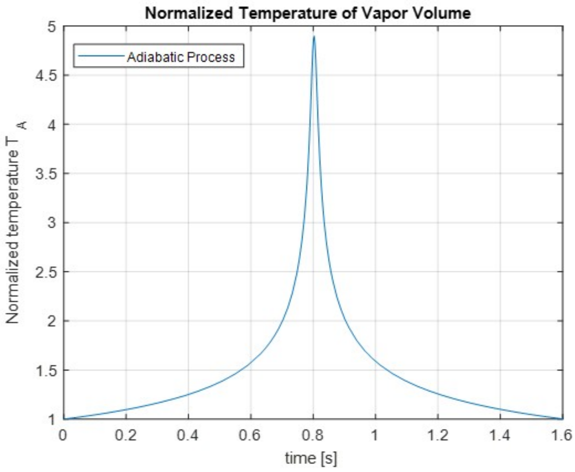


Figure 6: Normalized temperature

The thermal diffusive time scale for the liquid slug is $\tau_{diff,th} = \frac{L_{liquid}^2}{\alpha} \sim 10^5 s$, and it is much longer than the vapor-liquid energy exchange time $\Delta t = 0.8s$, and even longer than the duration when the liquid may obtain enough energy for vaporization. Therefore, the amount of evaporation in the first cycle is negligible, which justifies the previous assumption.

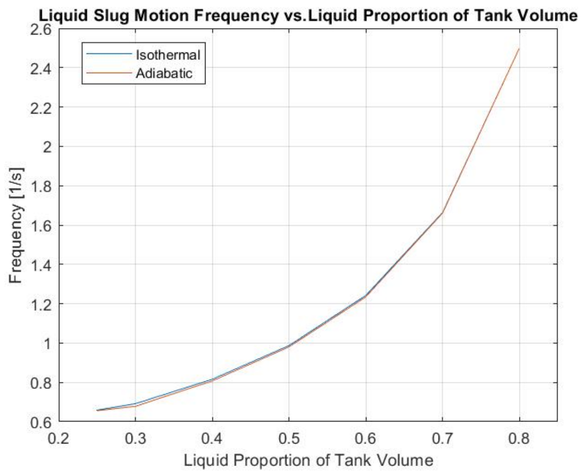


Figure 7: Frequencies with different liquid initial volume

Figure (7) shows the variation of liquid slug motion frequency at different percentage of the tank initially filled with liquid fuel, from approximately $0.7s^{-1}$ at 25% to $2.5s^{-1}$ at 80%. For both processes, the motion cycle frequency increases as higher portion of tank volume is filled with liquid. From 25% of tank length to 70%, isothermal process has a slightly higher frequency than adiabatic process; however, the two processes have the same frequency from 70% to 80%.

2. **Situation 2**
Consider a tank with the same geometry and liquid fuel with the same properties as in situation 1. Again, gasoline liquid fuel fills 25% of the total tank volume, but initially located in the middle of the tank. The tank moves at $U_{tank} = U_0 = 2\text{m/s}$ for $t < 0$, then it stops suddenly with $U_{tank} = 0\text{m/s}$ for $t \geq 0$, as shown below.

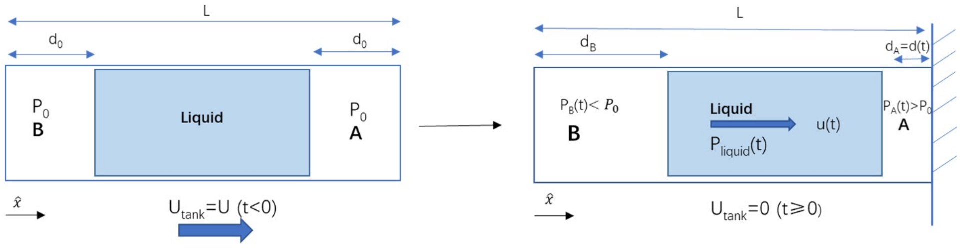
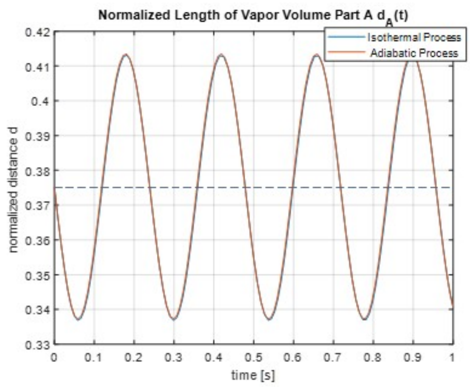


Figure 8: Schematic description of situation 2

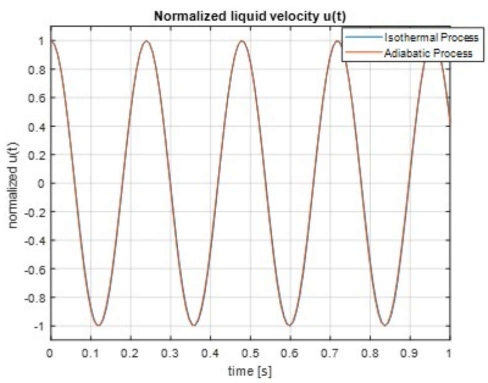
Because initially region A and B have the same volume and are in equilibrium, both sides are filled with the same amount of vapor, and thus, the Boyle's law is applicable, i.e. $\frac{P_A}{P_B} = \frac{V_B}{V_A}$ as in ODE (16). Parameters for calculation are listed in table (2).

P_0	ρ_l	U_0	$\tilde{d}_0$	γ
$10^5 Pa$	780 kg/m^3	2 m/s	0.375	1.3

Table 2: Calculation Parameters



(a) Normalized d(t)



(b) Normalized u(t)

Figure 9: Situation 2 with initial velocity $U_0 = 2$ m/s

Figure (9) shows that the liquid is moving back and forth around the central position periodically. The distance from central position to farthest it can reach is approximately $\Delta d = 0.077m$, and the time it takes to decelerate to zero velocity is approximately $\Delta t = 0.06s$. In this case, the Reynolds number is $Re_D = \frac{U_0 D}{\nu} = 1.67 \cdot 10^6$ with a turbulence flow entry length of $L_e = 1.359 Re_D^{1/4} \approx 48.8m \gg L = 2m$. The distance the liquid covers $\Delta d = 0.077m$, 0.16% of approximate entry length L_e. Therefore, the velocity profile change $u(r)$ with axial distance is negligible. The diffusive time scale $\tau_{diff} = \frac{R^2}{\nu} = 104167s$ is also much larger than Δt, the velocity profile change $u(r)$ with time development is negligible. Viscous effect is negligible in this case and axial velocity profile remains uniform in the entire cross-sectional area.

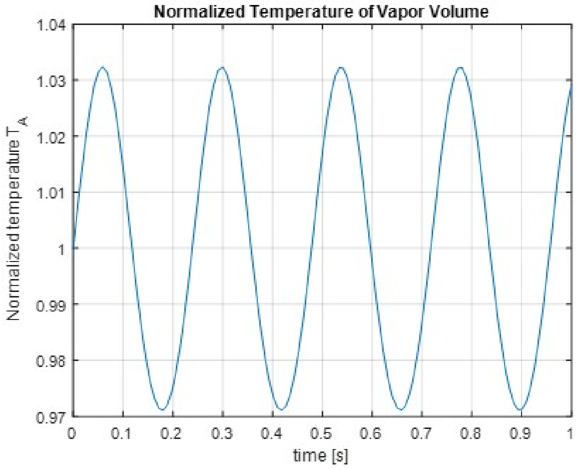


Figure 10: Normalized temperature

Similar to the time development of $d(t)$ and $u(t)$, the temperature also appears to develop in the shape of a sinuous wave with the same frequency. Unlike in case 1, the peak temperature now only reaches approximately $T_{max} \approx 1.03T_0 = 300K$; and the duration of energy exchange from vapor to liquid slug is even shorter, around $\Delta t = 0.06s$. Vaporization barely occurs in this case.

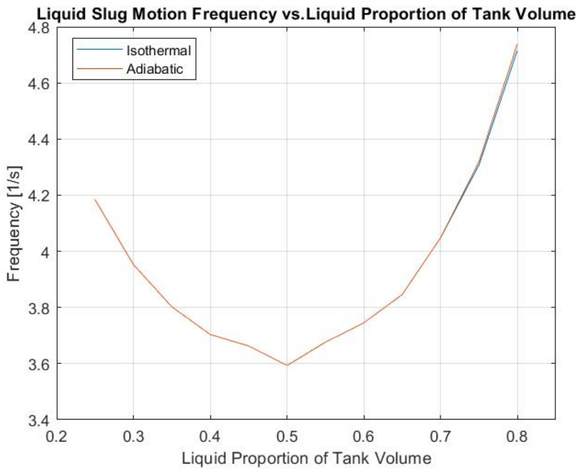


Figure 11: Frequencies with different liquid initial volume

In the case where the liquid slug is originally positioned in the middle, there exists a lowest motion frequency at $3.6s^{-1}$ when liquid fills 50% of the tank. Curves for the two processes mostly coincide with each other.

3. **Situation 3**
Consider a tank with the same geometry and liquid fuel with the same properties as previous. Again, gasoline liquid fuel fills 25% of the total tank volume, and initially located on the one side of the tank. The tank is stationary at $U_{tank} = 0\text{m/s}$ for $t < 0$, then it suddenly starts to move at $U_{tank} = U_0 = -2\text{m/s}$ for $t \geq 0$, as shown below.

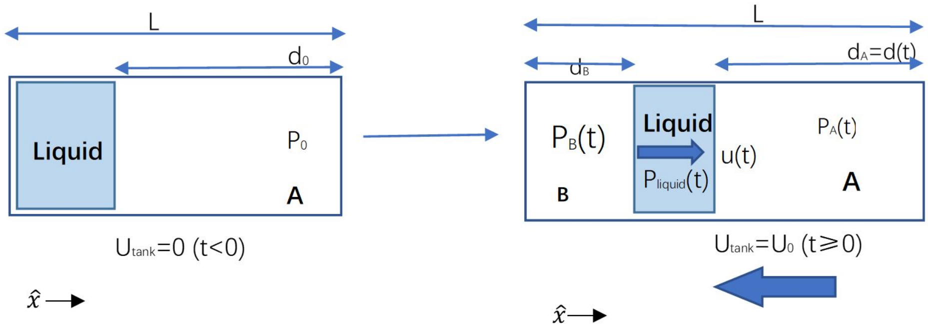
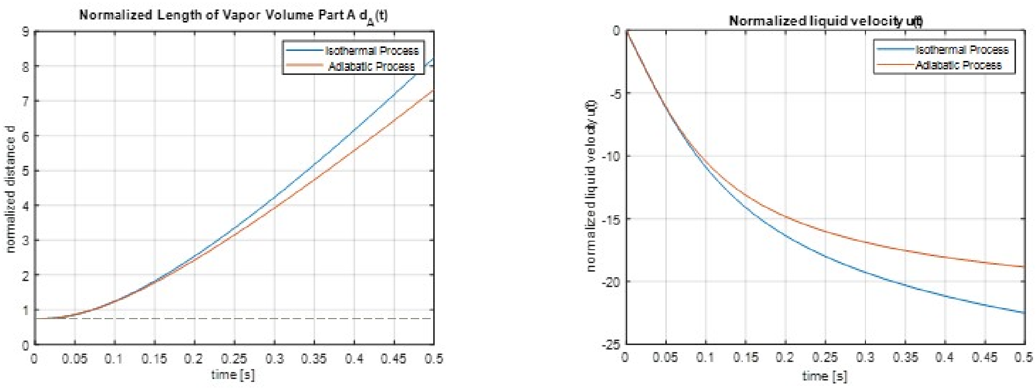


Figure 12: Schematic description of situation 3

Similarly as in case 1, initially there is no vapor volume in region B, Boyle's Law is inapplicable here. Assume that liquid evaporation in region B is negligible and pressure $P_B(t) = 0$ throughout the first cycle. The initial conditions are $\tilde{d}(0) = \tilde{d}_0$ and $\tilde{d}'(0) = -1$, whereas the difference lies in the relation between $u(t)$ and $d(t)$, as opposed to equation (4):

$$-\frac{\partial \tilde{d}}{\partial t} = \tilde{u}(t) + 1 \tag{18}$$

Solving ODE (15) with parameters in table (1):



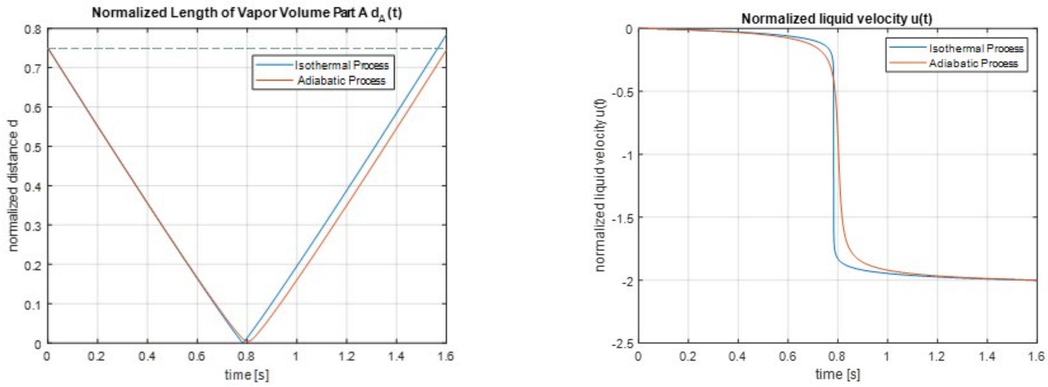
(a) Normalized d(t)

(b) Normalized u(t)

Figure 13: Situation 3 with initial velocity $U_0 = 2$ m/s

This result is not reasonable since $\tilde{d}_A(t) < 1$. The velocity of the tank $U_{tank}(t \geq 0) = U_0 = -2m/s$ is too low and the pressure in region A P_A is large enough to accelerate the liquid slug immediately to the tank velocity U_0, which results in both the tank and the liquid moving together at the same velocity, with the liquid slug always at the one end of the tank. Since the volume of region A vapor hardly changes, its temperature remains constant.

If U_{tank} is increased to $U_{tank}(t \geq 0) = U_0 = -100m/s$, P_A will not have been large enough to keep liquid slug and tank moving together immediately.



(a) Normalized d(t)

(b) Normalized u(t)

Figure 14: Situation 3 with initial velocity $U_0 = 100$ m/s

This result is only valid for the domain where $\tilde{d}(t) \leq 0.75$ since the liquid fuel is assumed to be incompressible. The liquid block first accelerates in the $-\hat{x}$ direction to some velocity $u(t_1 = 0.8037s) \approx -105.7m/s$, and reaches almost the other end of the tank, at approximately $\Delta t_1 = 0.8037s$. Up to this point, it has already accelerated to a velocity that is faster than the tank velocity ($|\tilde{u}(t_1)| > 1$). Vapor volume in region A V_A starts to increase and its pressure P_A starts to decrease. The liquid slug will achieve maximal velocity when it travels back to its original position where $\tilde{d}(t_2 \approx 1.6s) = 0.75$. The maximal velocity is $u_{max} = u(t_2 \approx 1.6s) \approx -200m/s$. After this, the solution is no longer valid, because during the first cycle, the evaporated vapor in region B becomes important, pushing the liquid slug to side A.

The Reynolds number $Re_D \sim 8.3 \cdot 10^7$ with $L_e \approx 129.8m \gg 0.75L = 1.5m$. The diffusive time scale $\tau_{diff} \approx 104167s \gg 0.8s$. Therefore, both the time and space developments have very little effect on the flow axial velocity profile, and thus the viscous effect is negligible.

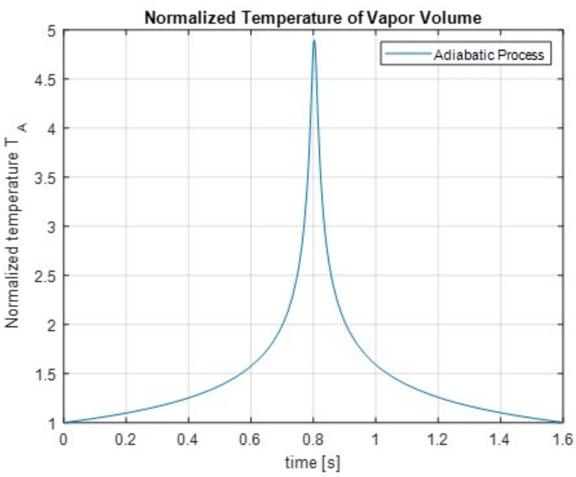


Figure 15: Normalized temperature

The vapor temperature development is almost the same as that in case 1 because the relative motion of liquid and tank in two cases are the same, and that temperature is a scalar. Again, vaporization is negligible in this case.

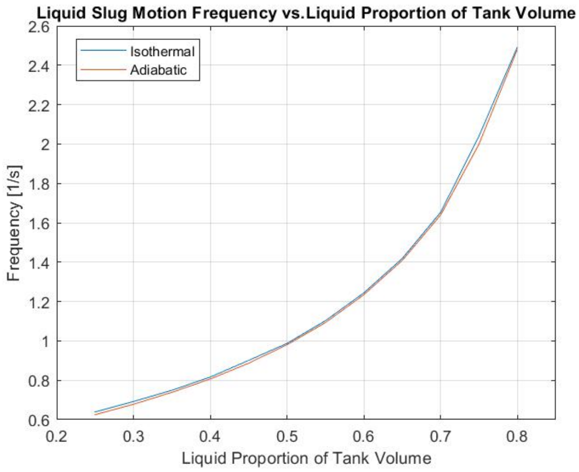


Figure 16: Frequencies with different liquid initial volume

Liquid slug motion frequency increases with liquid fraction, from $0.65s^{-1}$ at 25% volume to $2.5s^{-1}$ at 80% volume, similar to case 1.

4. **Section 4**
Consider a tank with the same geometry and liquid fuel with the same properties as previous and gasoline liquid fuel fills 25% of the total tank volume, but initially located in the middle of the tank. The tank is stationary $U_{tank} = 0$m/s for $t < 0$, then it suddenly starts to move at $U_{tank} = U_0 = +2$m/s for $t \geq 0$, as shown below.

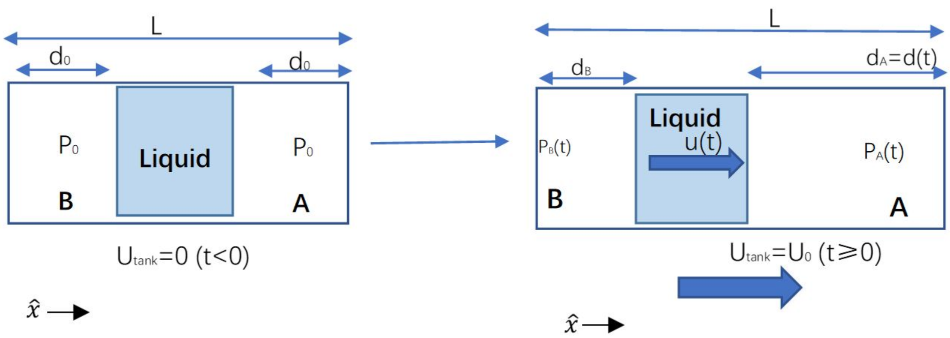
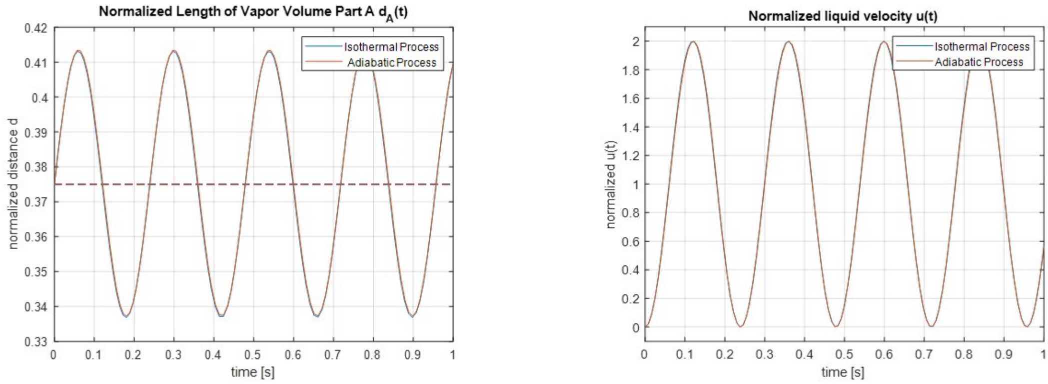


Figure 17: Schematic description of situation 4

Because initially region A and B have the same volume and are in equilibrium, both sides are filled with the same amount of vapor, and thus, the Boyle's law is applicable as in ODE (16). Parameters for calculation are listed in table (2). The relation between $d(t)$ and $u(t)$:

$$-\frac{\partial \tilde{d}}{\partial t} = \tilde{u}(t) - 1 \tag{19}$$

and the initial conditions are $\tilde{d}(0) = \tilde{d}_0$ and $\tilde{d}'(0) = +1$.



(a) Normalized d(t)

(b) Normalized u(t)

Figure 18: Situation 4 with initial velocity $U_0 = 2$ m/s

The results show that the liquid is moving back and forth around the central position periodically. The distance from central position to farthest it can reach is approximately $\Delta d \approx 0.076m$, and the time it takes to accelerate to tank velocity U_0 is $\Delta t \approx 0.057s$. In this case, the Reynolds number is $Re_D \sim 1.67 \cdot 10^6$ with a turbulence flow entry length of $L_e \approx 48.8m \gg \Delta d = 0.076m$. The diffusive time scale $\tau_{diff} = \frac{R^2}{\nu} \approx 104167s$ is also much larger than Δt. Therefore, the velocity profile change $u(r)$ with axial distance and time is negligible. Viscous effect is negligible in this case.

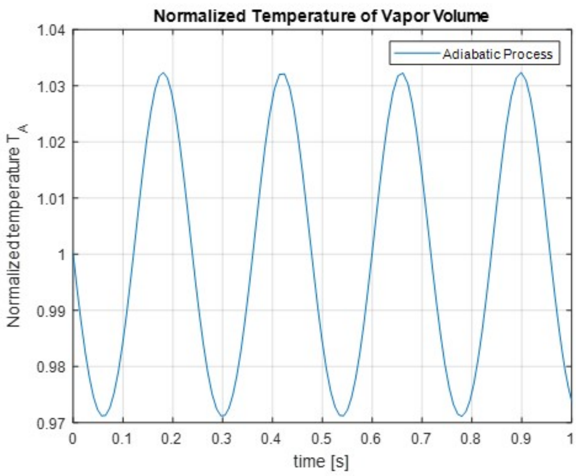


Figure 19: Normalized temperature

With the energy exchange time of $\Delta t \approx 0.06s$ and peak vapor temperature $T_{max} \approx 309K$, vaporization during the process can be neglected.

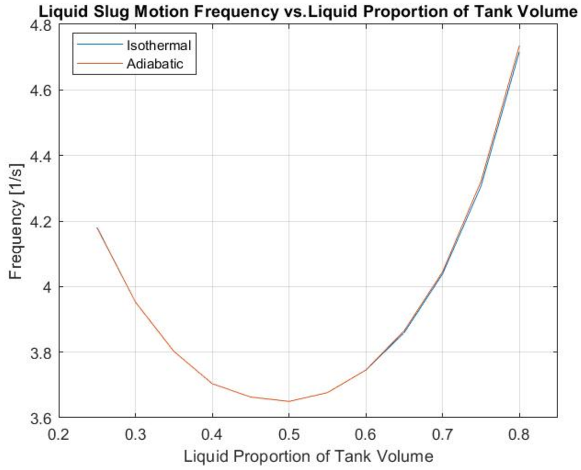


Figure 20: Frequencies with different liquid initial volume

Similar to case 2, the lowest liquid slug motion frequency, $3.65s^{-1}$, also occurs when 50% of the tank volume is filled with liquid fuel. Slight difference can be seen when the filled fraction is between 60% to 80%, and the adiabatic process has a higher frequency.

4 Conclusions

This research investigates the behavior of incompressible liquid fuel in a partially filled tank during spaceship acceleration and deceleration. Analytical models are set up at the beginning to associate liquid slug's equations of motions with thermodynamic properties of vapors. According to the liquid slug initial position within the tank, either the no-vapor assumption or Boyle's Law is applied to derive the ODE to solve. The solutions indicate some motion properties such as covered distance, velocity development and acceleration/deceleration time.

Final calculation results show that the differences between isothermal process and adiabatic process are very little. In all situations, the liquid slug motion duration is too short, and the tank length is too short for viscosity to come into effect in the first cycle. Therefore, during the liquid slug motion, its axial velocity profile change is negligible, and no boundary layer thickness needs to be taken into consideration, which justifies the inviscid assumption. Moreover, the energy transferred from vapor to liquid slug is not enough for evaporation to occur, which justifies the no-evaporation assumption previously made. With the justified assumptions, the liquid slug back-and-forth motion frequency is found to be a function of liquid filled fraction of the tank. It should be noticed that this study is only valid for the first cycle in situation 1 & 3 or first few cycles in situation 2 & 4 since evaporation will play a role in further liquid-vapor interactions.

Further work could include the effects of driven forces from the tank walls on the liquid slug by adding a perturbed term into liquid axial velocity and imposing corresponding boundary conditions at wall surfaces. A series of normal modes of perturbation can thus be calculated, some of which may decay with time due to viscous shear damping; while others may be excited to resonance depending on the liquid slug motion frequency. The local liquid axial velocity is closely associated with local vapor thermodynamic properties; therefore, stability of the entire vapor-liquid system can be analyzed. Other considerations may be further investigated as well, for example, the effects of tank inclination, the rotation of the tank and micro-gravity.

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