Research Project

Analytical investagtion of Faraday waves in Hele-Shaw cells

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In this study we investigate an analytical model of nonlinear Faraday waves in a Hele-shaw cell. This configuration bounds the flow between two parallel flat plates separated by a small gap, effectively reducing the dimensions of the problem, and allowing two-dimensional like flows to be explored experimentally and theoretically.

In 1831, Michael Faraday discovered that if a layer of liquid is placed on top of a vertically oscillating piston, a pattern of standing waves appears which oscillates at half the driving frequency. Although Faraday waves are well-known and comprehensively studied, recent discoveries continue to present new findings. In 2011, Rajchenbach [4] observed two types of non-linear standing waves with patterns of even and odd symmetry, through parametric excitation of water waves in a Hele-Shaw cell. Since two-dimensional odd standing waves had never been reported before, it drew particular interest.

In contrast to Faraday waves in unbounded flows, in a Hele-Shaw cell one must address the complexity of the capillary motion arising from the small scale of the gap size between the two close walls. Such a hydrodynamic model was suggested by Hamraoui [2]. In addition, Rajchenbach [4] did not address the effects of the surface tension and a more sophisticated model was developed by Li [3], taking this into account. To analyze this model, we used a perturbation technique of multiple-scale analysis to derive an amplitude equation that will describe the dynamics of the waves.

Our purpose in this project is to provide a detailed mathematical formulation for the nonlinear model that had been reported by Rajchenbach [4]. The project at hand aims at gaining a better understanding of the Hele-Shaw models that were demonstrated by Li [3], and to review fundamentals studies of Faraday waves such as Benjamin and Ursell [1].

References

[1] Thomas Brooke Benjamin and Fritz Joseph Ursell. The stability of the plane free surface of a liquid in vertical periodic motion. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, 225(1163):505–515, 1954.

[2] Ahmed Hamraoui, Krister Thuresson, Tommy Nylander, and Vassili Yaminsky. Can a dynamic contact angle be understood in terms of a friction coefficient? *Journal of colloid and interface science*, 226(2):199–204, 2000.

[3] Jing Li, Xiaochen Li, and Shijun Liao. Stability and hysteresis of faraday waves in hele-shaw cells. *Journal of Fluid Mechanics*, 871:694–716, 2019.

[4] Jean Rajchenbach, Alphonse Leroux, and Didier Clamond. New standing solitary waves in water. *Physical review letters*, 107(2):024502, 2011.