Trajectory simulation and investigation of uncertainty effects on a three-stage rocket designed by Rafael.

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Introduction

Black Falcon is a new rocket which was designed by *Rafael*. It has no controllers, meaning that once it was launched, nothing could regulate or stop it. This creates many possible dangerous situations - for example, one can only imagine what would happen if the rocket would go back during the flight, and instead of going to the ocean, it would go to a city full of citizens. As a result, it is essential to study the possible trajectories, and search for dangerous ones. Due to the importance of the matter, it was decided that two independent groups would study the possible trajectories and hopefully, in the end, the two groups will get to similar and safe results. This paper sums up the research our group has done in the *Technion*.

To be more specific, our work on finding dangerous trajectories was based on a 6DOF simulation with various uncertainties. such as, non axial thrust, wind disturbances, aerodynamic coefficients uncertainties, and broken wings configurations (at the launching moment, some of the wings could break).

General Information

In this section we will describe the full information and the method of solution, to have the base of understanding this paper.

Rocket stages

Black Falcon has 3 rocket stages. The first stage is consisted of an ordinary rocket engine. According to *Rafael* studies, the burnout time of this stage is 3.235 seconds. After this stage is over, a screw in the head of the rocket is broken and this lets the air go inside the engine - thus creates the second stage which is the ramjet stage. Finally, the last stage is the rocket's free flight phase where there is no thrust. Unfortunately, due to many constraints, it was not possible to model the second stage, and therefore we considered our rocket to be only a two-stage rocket (with the first and last stages).

Rocket coordinate system

The rocket's coordinate system is considered to be a classical body coordinate system as shown in Figure 1. Furthermore, the earth coordinate system is taken to be - X axis goes west, Y axis goes north and Z axis goes to earth.

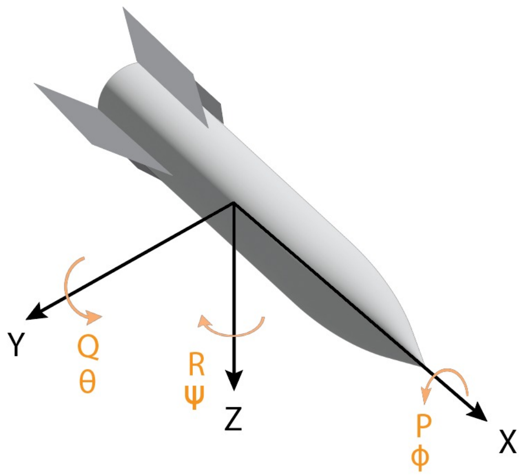


Figure 1: body axes

Method

Firstly, we will create a 3DOF simulation to get the trajectory of this rocket. Secondly we will do the same with a 6DOF simulation. The 3DOF is of course way less accurate, but it is important to create it so that we can compare between the two simulations and see if our 6DOF is not too far from the 3DOF - if it is, then our simulation is incorrect. Afterwards, we will simulate non axial thrust to the engine - to see it's effect on the trajectory. In addition, we will calculate and imply the aerodynamic of different broken wings configurations into the 6DOF simulation. And finally, we will create *Monte Carlo* simulations for a given wind profile and see if the trajectories are safe - this wind profile is the one expected to be at july. (At the time in which the experiments where scheduled)

Data taken from ***Rafael*** and ***Yanat***

For our study we will use *Rafael*'s experiments data which consists of:

- *txt* file with known thrust, weight and center of gravity location for every 0.05 seconds at the first stage.
- *Excel* file with known C_d as function of the Mach number (M_{0i}) in all of the stages.
- Moments of inertia at the beginning of each stage.
- Reference area of the rocket.
- The initial conditions of the rocket.
- wind tunnel experiments for different configurations of broken wings.

From *Yanat* we were able to get the wind profile for july.

The 3DOF simulation

Our first simulation for the rocket will be coded in *Matlab*, and will be a 3DOF simulation (3 transition directions - XYZ). We defined the vector of our variables as:

$$\vec{X} = \begin{Bmatrix} V \\ \gamma \\ \chi \\ X \\ Y \\ Z \end{Bmatrix}$$

We will use the Euler method, namely - $\vec{X}_{i+1} = \vec{X}_i + \dot{\vec{X}}_i \cdot dt$. where i is the number of the iteration, and dt is the time step of the simulation. As provided by *Rafael* the initial conditions are:

$$\vec{X}_0 = \begin{Bmatrix} 40\ \frac{m}{sec} \\ 30^\circ \\ 0^\circ \\ 0\ m \\ 0\ m \\ 0\ m \end{Bmatrix}$$

Equations of motion

In order to use the Euler method we need a way to describe $\dot{\vec{X}}$. That is where the equations of motion help us:

$$\dot{\vec{X}} = \begin{Bmatrix} \dot{V} \\ \dot{\gamma} \\ \dot{\chi} \\ \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix} = \begin{Bmatrix} g[\frac{T\cos(\alpha+\epsilon)-D}{W} - \sin\gamma] \\ \frac{g}{V}[\frac{T\sin(\alpha+\epsilon)+L}{W}\cos\mu - \cos\gamma] \\ \frac{g\sin\mu}{V\cos\gamma} \cdot \frac{T\sin(\alpha+\epsilon)+L}{W} \\ V\cos\gamma\cos\chi \\ V\cos\gamma\sin\chi \\ -V\sin\gamma \end{Bmatrix}$$

Where α is the angle of attack, β is the sideslip angle and X, Y, Z are the rocket position with respect to a starting point on earth. g is the gravitational constant which is taken as $g = 9.81[\frac{m}{s^2}]$. V is the speed, L, D are the lift and drag, respectively. W is the gravity force. χ, μ are the yawing and spinning angles respectively. γ is the flight path angle. ϵ is a parameter of non axial thrust. Now, let us make the following assumptions:

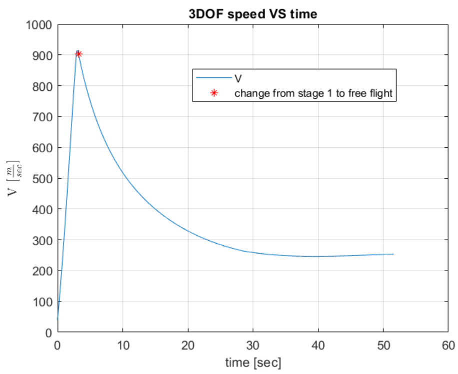
$$\begin{Bmatrix} \alpha = \dot{\alpha} = 0 \\ \beta = \dot{\beta} = 0 \\ \mu = \dot{\mu} = 0 \\ D = \frac{1}{2}\rho V^2 S C_d \\ \epsilon = 0 \\ L = 0 \\ W = mg \end{Bmatrix}$$

These assumptions give us:

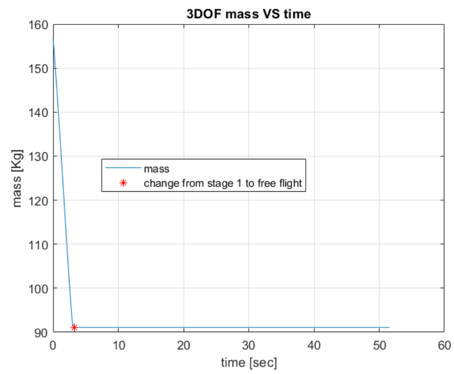
$$\dot{\vec{X}} = \begin{Bmatrix} \dot{V} \\ \dot{\gamma} \\ \dot{\chi} \\ \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix} = \begin{Bmatrix} g[\frac{T-\frac{1}{2}\rho V^2 S C_d}{mg} - \sin\gamma] \\ \frac{g}{V}\cos\gamma \\ 0 \\ V\cos\gamma\cos\chi_0 \\ V\cos\gamma\sin\chi_0 \\ -V\sin\gamma \end{Bmatrix}$$

The time steps at each iteration are $dt = 0.01$ seconds (we saw that for smaller steps the change in the results is practically zero). The reference area of the rocket is given by $S = \frac{\pi}{4}D^2$ where D is the diameter of the rocket and it was given by *Rafael*. The mass and the thrust were linearly interpolated using the tables of mass and thrust as function of time that were given by *Rafael*. Furthermore, we used 1976 *COESA* atmospheric model in order to get the speed of sound (a) and the density of the air (ρ) as function of the height. Using a allowed us to calculate the Mach number by $M = \frac{V}{a}$. Only then we could use the table of Mach VS C_d from *Rafael* in order to interpolate C_d. Under our assumption the yawing angle is constant and equal to χ_0. We claim that at the beginning of the flight this angle is zero, in other words $\chi_0 = 0$. This gives as $\dot{Y} = 0$, meaning the rocket moves only in the XZ plane.

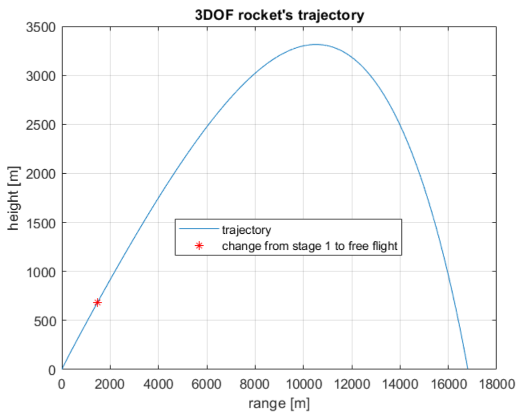
The results of this simulation are shown in Figure 2. In Figure 2(a) we can see that the speed's slope is positive only when there is thrust (stage 1), but in the last few moments of stage one, the velocity decreases - meaning that the thrust is lower than all the other forces in these few moments. After the engine stops working, the speed gets smaller until we get to the terminal speed (where drag and gravity cancel each other). In Figure 2(b) we can see the mass profile against the time. In the first stage we can see that the mass is linearly interpolated and in the free flight the mass is constant - as expected. In Figure 2(c) we see the full trajectory of the rocket, with the exact location of the change from stage 1 to the free flight.



(a) 3DOF speed Vs time



(b) 3DOF mass VS time



(c) 3DOF rocket's trajectory

Figure 2: 3DOF simulation results

The 6DOF simulation

In order to have reliable simulation results, 3DOF simulation alone is not sufficient. Therefore a 6DOF simulation is the heart of the research. In this simulation the rocket is allowed to move in the direction of each one the its axes, and rotate around each axes. The simulation was created in *Simulink* and it's core is the "6DOF (Euler angles)" block which can be seen in Figure 3. An important note would be that inside this block it was assumed that all the fuel which was burned is used to generate thrust, without any losses. Moreover, this block does consider the change in moments of inertia with time.

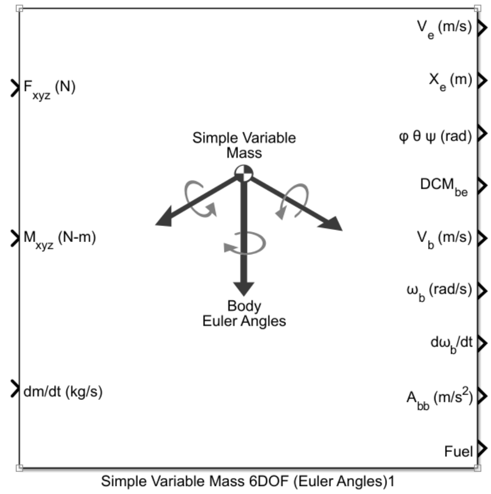


Figure 3: simple variable mass 6DOF Euler Angles block

As provided by *Rafeal*, we have the moments of inertia I_{xx}, I_{yy}, I_{zz} at the beginning and the ending of each stage. Therefore it was simple to imply it in our simulation. The products of inertia are taken as zero due to the rocket's symmetry. The mass of the rocket is given as a function of time in the first stage (in a table). As a result, the empty and the full mass of the aircraft is quite easy to obtain. The derivative of the mass with respect to time is also required for the simulation. This derivative was derived by fitting a 5th degree polynomial to the mass vs time curve. The fitting can be observed in Figure 4, where it can be seen that the fitted curve is pretty accurately representing the curve that is plotted using the given table. This analysis process was made only for the first stage. For the free flight stage, it is obvious that the mass is constant and it's derivative is zero.

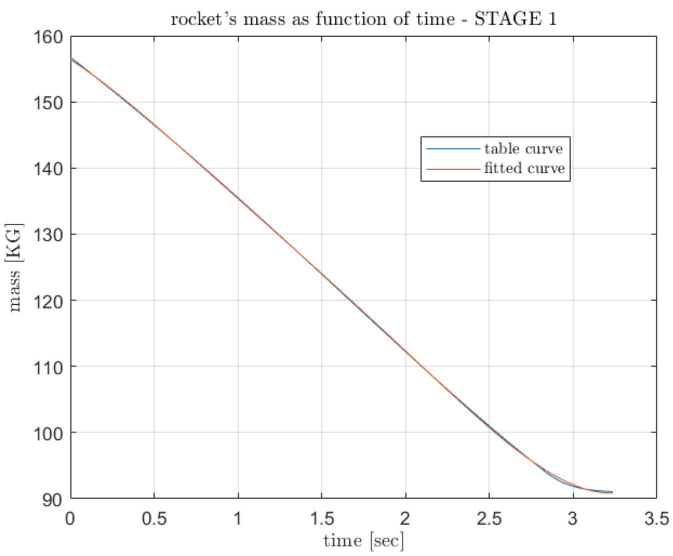


Figure 4: mass vs time - table and fitted curves

Moreover, the initial conditions of the rockets were set to satisfy *Rafeal*'s demand - initial speed of 40 meters per second and initial pitch angle of 30° . Furthermore, using the 6DOF block it was possible to get the rocket's speed and position in both body and earth axes. Using this data and the 1976 COESA atmospheric model it was possible to obtain $\rho, a, Mach$. in addition, γ was calculated using the known relation of $\sin^{-1}\left(\frac{\dot{h}}{V}\right)$, where $\dot{h}$ is the derivative of the height with respect to time. α, β were calculated using the equations:

$$\alpha = \tan^{-1}\left(\frac{w}{u}\right)$$
$$\beta = \sin^{-1}\left(\frac{v}{V}\right)$$

Where u, v, w are the components of the speed in body axes and V is the the rocket's speed absolute value. Moreover, the simulation condition to stop is when the rocket is hitting the ground, and the steps in time were taken similarly to the 3DOF simulation as $dt = 0.01$ seconds.

Forces transformation from wind to body axes

Because the *Simulink*'s 6DOF block is in the body axes, it requires the forces and moments that act on the body in body axes. The thrust T is acting in body axes, and the gravity force can be obtain in body axes by multiplying it with the DCM matrix from the left. However the aerodynamic forces (lift and drag) are defined in the wind axes. In this section we will explain how they were transformed to body axes.

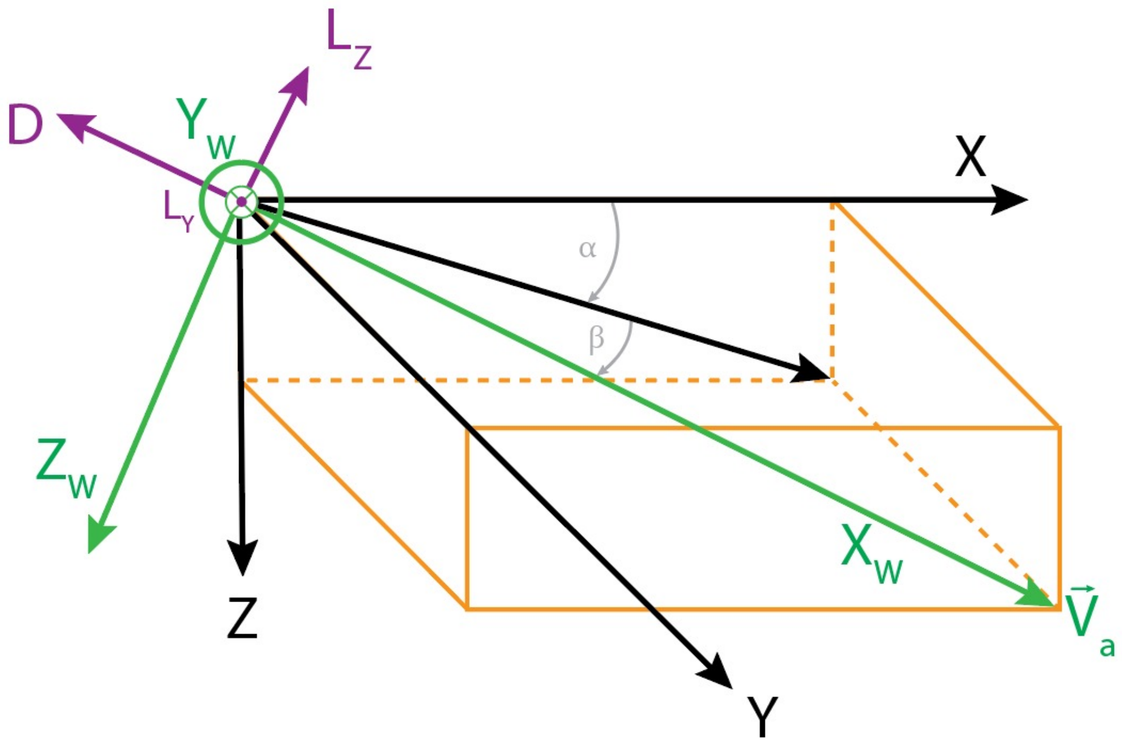


Figure 5: aerodynamic forces transformation from wind to body axes

In Figure 5 the axes X, Y, Z and X_w, Y_w, Z_w represent the body axes and the wind axes respectively. Moreover, the rocket with the speed of $\vec{V_a}$ has angle of attack and sideslip angle (α, β). Furthermore it is influenced by the lift forces (L_Z, L_Y), and the drag force D. We wish to evaluate the total force vector in body axes (F_b). Firstly, the forces in the wind axes can be written such that:

$$F_v = \begin{bmatrix} -D \\ -L_y \\ -L_z \end{bmatrix}$$

Secondly, it can be transformed to body axes using rotation matrices around Z_w and Y_w in the following manner:

$$
\begin{aligned}
F_b &= \begin{bmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \begin{bmatrix} \cos(-\beta) & \sin(-\beta) & 0 \\ -\sin(-\beta) & \cos(-\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -D \\ -L_y \\ -L_z \end{bmatrix} \\
&= \begin{bmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -D \\ -L_y \\ -L_z \end{bmatrix} \\
&= \begin{bmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \begin{bmatrix} -D\cos(\beta) + L_y\sin(\beta) \\ -D\sin(\beta) - L_y\cos(\beta) \\ -L_z \end{bmatrix} \\
&= \begin{bmatrix} \cos(\alpha)\left(-D\cos(\beta) + L_y\sin(\beta)\right) + L_z\sin(\alpha) \\ -D\sin(\beta) - L_y\cos(\beta) \\ \sin(\alpha)\left(-D\cos(\beta) + L_y\sin(\beta)\right) - L_z\cos(\alpha) \end{bmatrix} \\
&= \begin{bmatrix} -D\cos(\beta)\cos(\alpha) + L_y\sin(\beta)\cos(\alpha) + L_z\sin(\alpha) \\ -D\sin(\beta) - L_y\cos(\beta) \\ -D\cos(\beta)\sin(\alpha) + L_y\sin(\beta)\sin(\alpha) - L_z\cos(\alpha) \end{bmatrix}
\end{aligned}
$$

This force vector can be calculated in our simulation. α, β are given in the 6DOF block, and the drag force can be calculated in the same manner as it was calculated in the 3DOF simulation. Furthermore, the lift forces can be obtained using:

$$
\begin{aligned}
L_z &= \frac{1}{2}\rho V^2 S C_{lz} = \frac{1}{2}\rho V^2 S \left(C_{l0} + C_{l,\alpha}\alpha\right) \\
L_y &= \frac{1}{2}\rho V^2 S C_{ly} = \frac{1}{2}\rho V^2 S \left(C_{l0} + C_{l,\beta}\beta\right)
\end{aligned}
$$

Where, in this part of the project the aerodynamic coefficients where not yet given by *Rafeal*. So for a first evaluation C_{l0} was taken as zero, and $C_{l\alpha}, C_{l\beta}$ where taken to be similar to a different rocket. Later, in the modified 6DOF simulation, these coefficients will be taken according to given data.

Moments calculation

In this section, we will discuss how the moments were calculated. The reason body axes where chosen to represent the forces and moments were due to the simplicity of the moments calculation in these axes:

$$
M_b = \frac{1}{2}\rho V^2 S d \begin{bmatrix} \frac{d}{V} C_{m_p} p \\ \frac{d}{V} C_{m_q} q + C_{m_\alpha}\alpha \\ \frac{d}{V} C_{m_r} r + C_{m_\beta}\beta \end{bmatrix}
$$

Where d is the diameter of the rocket, and p, q, r are the rocket's rotation rates (known from the 6DOF block). In addition, $C_{m_p}, C_{m_q}, C_{m_r}, C_{m_\alpha}, C_{m_\beta}$ were taken to be similar to a different rocket (because these coefficients were not yet given to us). Later, in the modified 6DOF simulation, these coefficients will be taken according to given data.

Forces and moments caused by nonaxial thrust

In this section, we will discuss how non axial thrust influences the calculations of the forces and moments.

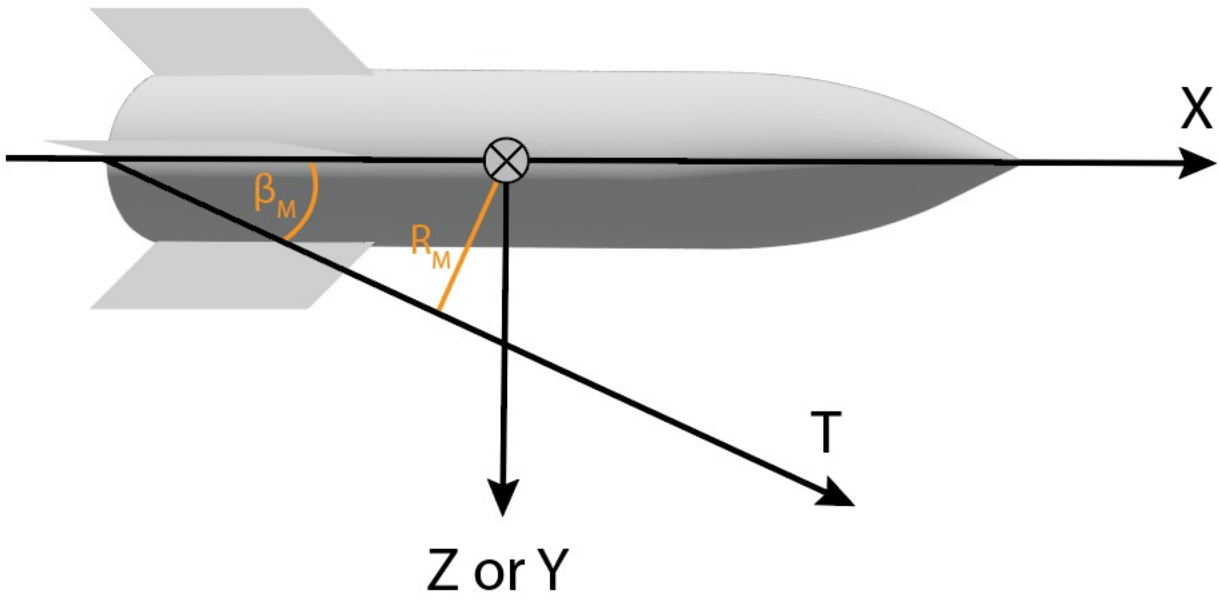


Figure 6: non axial thrust demonstration

In Figure 6 we can see how non-axial thrust looks like - it can be a picture in the XZ plane (where X is to the right and Z is down), or it can be a picture in the XY plane (where X is to the right and Y is down). Non axial thrust has components in both planes, such that in the XZ plane we have $R_M = R_{m\theta}, \beta_m = \beta_{m\theta}$, and in the XY plane we have $R_M = R_{m\psi}, \beta_m = \beta_{m\psi}$. In order to calculate the thrust in body axes (T_b), it was assumed that the non axiality influences only on the moments, and not the forces (due to the assumption that the non axial angles are very small). Therefore the trust vector in body axes will look as:

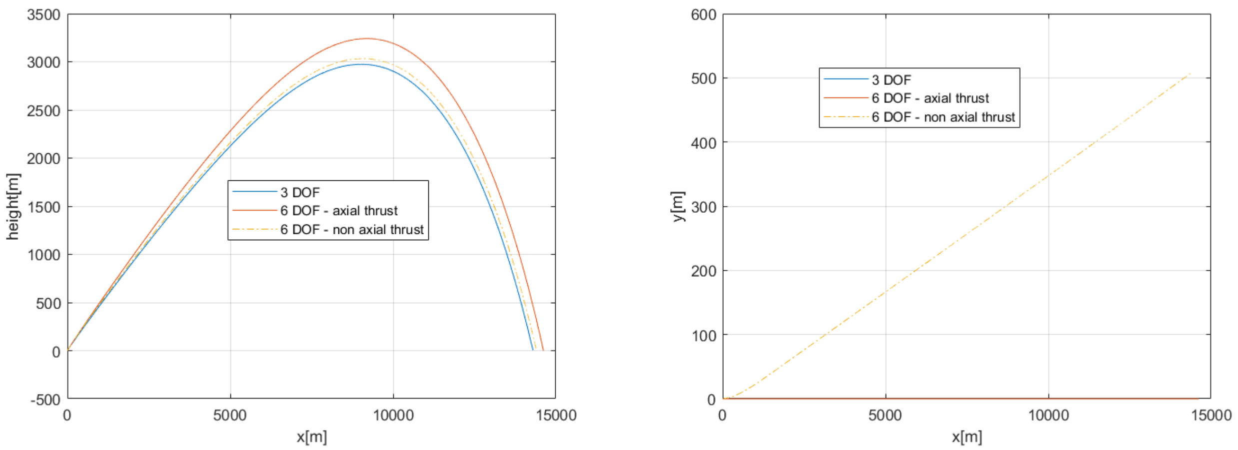
$$T_b = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix}$$

Moreover, we wish to obtain the moments caused by the non axial thrust. So after assuming very small non axial angles, we got:

$$\vec{M}_T = \begin{bmatrix} 0 \\ R_{m\theta} \cdot T \\ R_{m\psi} \cdot T \end{bmatrix}$$

6DOF simulation's results

After describing how the forces, moments, and mass derivative were obtained, we can run the simulation. In Figure 7 the results are demonstrated including the 3DOF and 6DOF simulations for axial thrust, and the 6DOF simulation for non axial thrust (in the case of $R_{m\psi} = R_{m\theta} = 10^{-3}$ Meters, $\beta_{m\psi} = \beta_{m\theta} = 1°$). It is important to notice how similar in shape are the results of the axial thrust in both the 3DOF and the 6DOF simulations (the results in the XZ plane). In addition, because $\beta_{m\psi} > 0$ we can see positive deviation in the Y direction in Figure 7(b).



(a) height Vs x of 3DOF and 6DOF

(b) y Vs x of 3DOF and 6DOF

Figure 7: 6DOF and 3DOF trajectories

Forces and moments of several broken wing configurations

One of our main interests in the project is to discuss how some configurations of broken wings affect the rocket's trajectory. This is important to investigate due to the known problems which appear once a wing gets broken. In this section, we will discuss how to calculate the forces and moments acting on the rocket in different configurations of broken wings. In figure 8 the back view of the rocket is presented with the number of each one of the four wings.

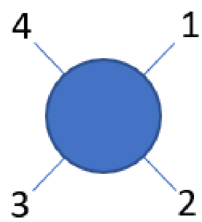


Figure 8: back view of the rocket - numbering the wings

In this part of the work, *Rafael* had provided us with more data derived from wind tunnel experiments. This data includes graphs for different configurations of broken wings. These are graphs for $C_Z^\alpha, C_M^\alpha, C_N^\alpha, C_Y^\alpha, C_R^\alpha$ as function of α. Where C_Z^α, C_Y^α are the forces coefficients in the Z and Y axes respectively, and $C_m^\alpha, C_R^\alpha, C_n^\alpha$ are the moment coefficients for pitching, rolling and yawing respectively. However, these graphs are given for only two Mach numbers (for $M = 0.8, 2.4$). To overcome this problem it was decided to have a linear interpolation of these coefficients as function of the Mach number, such that, for Mach numbers below 0.8 the coefficients of $M = 0.8$ will be taken, and for Mach numbers above 2.4 the coefficients of $M = 2.4$ will be taken, and in between 0.8 and 2.4, linear interpolation of the coefficients will be taken.

To simplify the problem, we have decided to take linear formulation for each of these coefficients. The linear formulation is around $\alpha = 0°$, and is clearly valid only for small angles of attack. In addition, the slopes of the linear formulations for each Mach number were obtained manually (by finding two points on the graphs, and finding the slope around $\alpha = 0°$), and can be observed in both Table 1 and Table 2.

case number	Configuration	$C_{Z,\alpha}$	$C_{M,\alpha}$	$C_{N,\alpha}$	$C_{Y,\alpha}$	$C_{R,\alpha}$
1		-14.33	-31.83	0	0	0
2		-11.46	-20.05	-14.32	3.294	1.755
3		-6.68	0	0	0	3.82
4		-5.73	3.183	-11.46	2.58	2.256
5		-10.51	-16.867	-25.78	5.874	0

Table 1: slope coefficients for $M = 0.8$

Where configuration number 5 in Table 1 was not tested in the wind tunnel. Its results were derived by superposition of different results. The superposition includes summing configurations 2 and 4 and subtracting configuration 3, this concept is demonstrated in Figure 9.

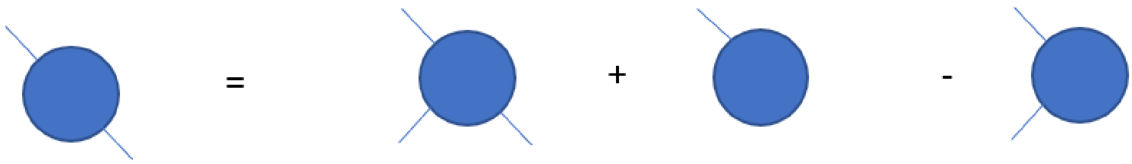


Figure 9: The concept of obtaining broken wings 1 and 3 configuration for Mach 0.8

In the same manner, configuration number 6 in Table 2 was not tested in the wind tunnel. Its results were derived by superposition of different results. The superposition is demonstrated in Figure 10.

Figure 10: The concept of obtaining broken wings 1 and 2 and 3 configuration for Mach 2.4

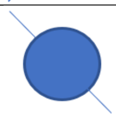
case number	Configuration	$C_{Z,\alpha}$	$C_{M,\alpha}$	$C_{N,\alpha}$	$C_{Y,\alpha}$	$C_{R,\alpha}$
1		-13.13	-17.47	0	0	0
2		-10.53	-7.5	-10.03	2.19	1.62
3		-7.88	4.58	0	0	3.72
4		-7.88	4.01	-20.05	4.3	0
5		-9.67	-1.24	0	0	0
6		-5.23	16.09	-10.02	2.3	1.62

Table 2: slope coefficients for $M = 2.4$

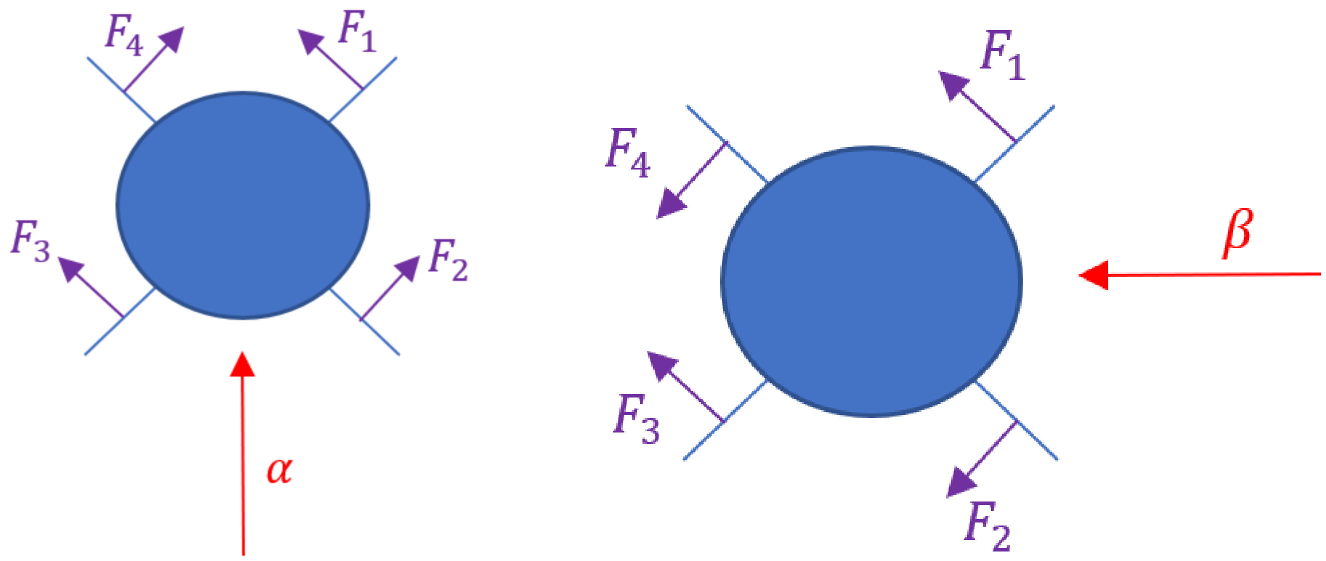
The forces and moments which are caused by α can be easily calculated from the coefficients $C_Z^\alpha, C_M^\alpha, C_N^\alpha, C_Y^\alpha, C_R^\alpha$. However, there are forces and moments which are caused by β, let us discuss how they were calculated: Firstly, we have assumed that the forces that act on the wings are only perpendicular to the wings. In Figure 11(a) we can see how these forces look like with an angle of attack, and in Figure 11(b) we can how these forces look like with an sideslip angle. significantly, in Figure 11, a symmetry can be observed between the angle of attack problem and the sideslip problem. If we analyse the two problems with different configurations of broken wings, we will see that due to symmetry, the following relations can be obtained:

$$
\begin{aligned}
C_Z^\alpha &= C_Y^\beta \\
C_M^\alpha &= -C_N^\beta \\
C_N^\alpha &= -C_M^\beta \\
C_Y^\alpha &= C_Z^\beta \\
symm_{factor} \cdot C_R^\alpha &= C_R^\beta
\end{aligned}
$$

As a result it is easy to obtain $C_Z^\beta, C_M^\beta, C_N^\beta, C_Y^\beta, C_R^\beta$ from the tables and the symmetry relations. And from these coefficients the forces and moments which are caused by β can be easily calculated. To sum up, the total forces and moments will be a superposition between those who are caused by α, and those who are caused by β.

Note that the variable $symm_{factor}$ changes with the geometry of the configuration. To be specific when looking at the configurations specified in Table 1 (as the case number column):
For case number 3, $symm_{factor} = 0$.
For case number 4, $symm_{factor} = -1$.
For case numbers 1,2,5, $symm_{factor} = 1$.

And when looking at the configurations specified in Table 2 (as the case number column):
For case number 3, $symm_{factor} = 0$.
For case number 6, $symm_{factor} = -1$.
For case numbers 1,2,4, $symm_{factor} = 1$.
For case number 5, $C_R^\alpha = 0$, but C_R^β will be the same as C_R^α in case 3 in Table 1



(a) forces demonstration on wings with an angle of attack (b) forces demonstration on wings with a sideslip angle

Figure 11: forces demonstration on wings with angle of attack and sideslip angle

The modified 6DOF simulation

In this section, we will generalized the previous 6DOF simulation to contain some broken wing configurations with non axial thrust and wind disturbances. Furthermore, this 6DOF model will be the one used in our *Monte Carlo* simulations.

Forces and moments calculations

The aerodynamic forces (except drag) and moments will be calculated for each broken wind configuration separately according to the previous section. Moreover, the thrust and drag forces will be calculated in the same manner as introduced in the previous 6DOF simulation.

Non axial thrust

The forces and moments which are caused by the non axial thrust will be calculated a bit differently compared to the previous non axial calculations. The basis of β_M, R_M which was introduced in Figure 6 remains the same, but due to *Rafael*'s request of characterizing the non axial thrust with only β_M, the following change was created:

It is obvious from Figure 6 that:

$$R_M = X_{cg} \sin(\beta_M)$$

Where X_{cg} is the distance of the center of mass from the back of the rocket. X_{cg} changes in the burning stages, but reaches a final and constant value of $1560mm$ at the beginning of the free flight stage. We will assume that $X_{cg} = 1560mm$ throughout the flight. This assumption is actually strict because this value is the maximum value of X_{cg}, and taking the maximum value of X_{cg} means taking the maximum value of R_M (which means taking bigger non axial parameters than in the actual case). Thus, from now on the non axial thrust will be function of only β_{m_θ} and β_{m_ψ}.

Wind disturbances

Wind disturbances are essential to consider when talking about rocket trajectories. In our work, we will discuss about one specific profile of wind disturbances:

A profile which was given to us by *Yanat*. This wind profile is specifically for july, due to that fact that the experiments of this rocket are planned to be on july.

Furthermore, we have been told that the launch will be with azimuth angle of 300° at the launching moment (meaning mostly to the west but with 25° deviation to the north). This is an important data, because the the wind disturbances will have different influences depending on the direction of the rocket at the moment of launch.

Validity check

To demonstrate the validity of the modified 6DOF simulation, one can take the case of non broken wings and check if the modified 6DOF simulation converges to the original 6DOF simulation. For the comparison, axial thrust, and zero wind disturbance were taken. This comparison can be observen in Figure 12. It can be seen that indeed the trajectory of the modified 6DOF simulation converges to the trajectory od the original 6DOF simulation, and this gives us confidence about the validity of the modified 6DOF simulation. In both simulations there were zero movements in the Y direction, due to the axial thrust and no wind disturbance.

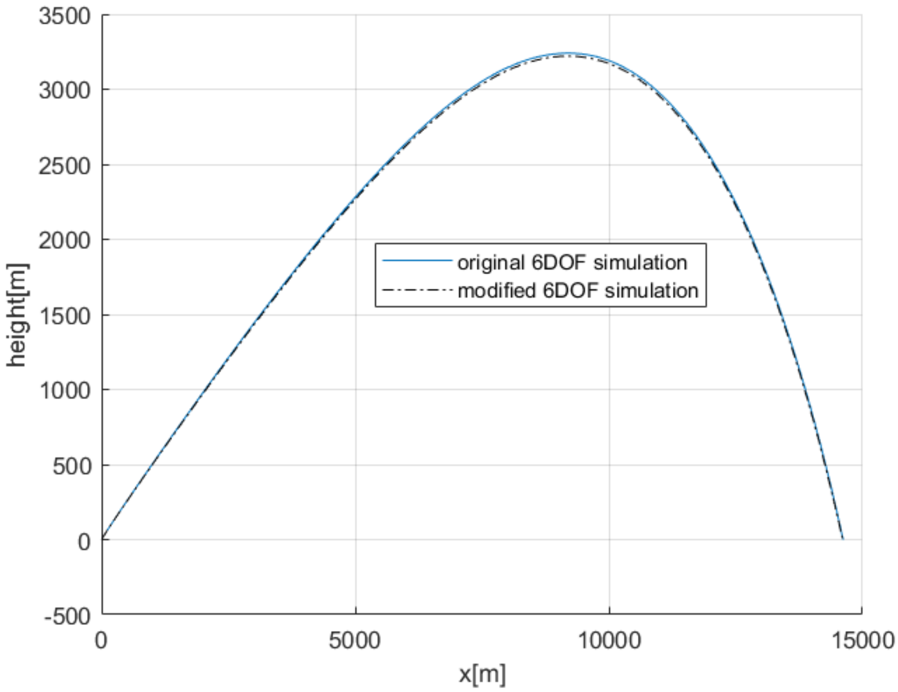


Figure 12: trajectory of the modified 6DOF Vs the original 6DOF

The *Monte Carlo* simulations

To consider all the possible uncertainties in the trajectory analysis, a *Monte Carlo* simulation was created. In this section, we will talk about the simulation for july specifically. For each configuration, we will draw non-axial thrust parameters between ± 3 miliradian. Meaning $\beta_{m\theta}$ and $\beta_{m\psi}$ are uniformuly distributed in the limits:

$$-3 \cdot 10^{-3} \leq \beta_{m\theta} \leq 3 \cdot 10^{-3}$$
$$-3 \cdot 10^{-3} \leq \beta_{m\psi} \leq 3 \cdot 10^{-3}$$

For this wind profile, we will use the data given by *Yanat*. This data is a table, and its columns are $h, E(wind_E), \sigma(wind_E), E(wind_N), \sigma(wind_N)$. meaning that for every height in the table, the wind from east to west is normally distributed with the mean of $E(wind_E)$ and the standard deviation of $\sigma(wind_E)$. The same goes for the wind from north to south. In the *Monte Carlo* simulations, after drawing the values of the winds for every height, we will linearly interpolate over the values of the height in the table.

In addition, there are uncertainties on the tip-off rates of yawing and pitching at the launching moment. And their mean and standard deviation were given by *Rafael*. These uncertainties are modeled in our *Monte Carlo* simulation.

Furthermore, an uncertainty on the aerodynamic coefficients was inserted to the simulation, Such that, in the *Monte Carlo* simulations the aerodynamic coefficients will be multiplied by a uniformly distributed factor which can change the coefficients' values by $\pm 30\%$.

Moreover, for each configuration, the *Monte Carlo* simulation consists of many 6DOF simulations. And because the aerodynamic coefficients were developed as linear functions of α, β, it is obvious that our simulations will not be valid for the cases of big α, β. As a result, we have decided to model the aerodynamic for high angles as saturation model. Meaning that if the angles exceed $\pm 20°$, then we will take the values for $\pm 20°$ accordingly.

Case of none wings broken

This case is shown in Figure 13.

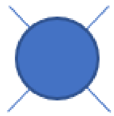
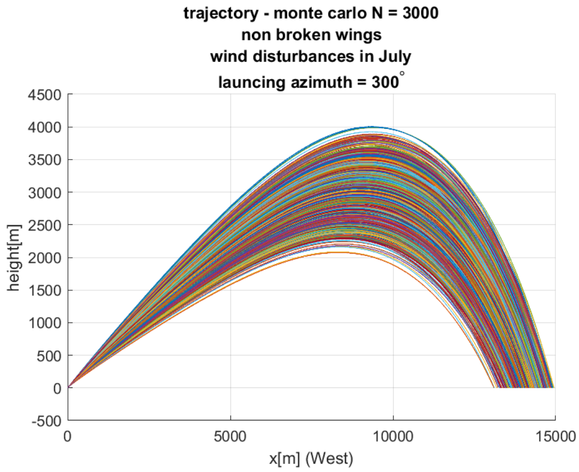
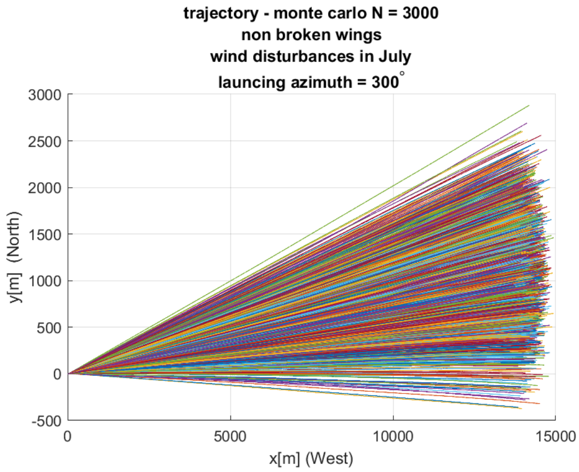


Figure 13: None broken wings configuration

When running the *Monte Carlo* simulation the following results were obtained:



(a) height Vs x none broken wings in july



(b) y Vs x non broken wings in july

Figure 14: *Monte Carlo* trajectories for non broken wings in july

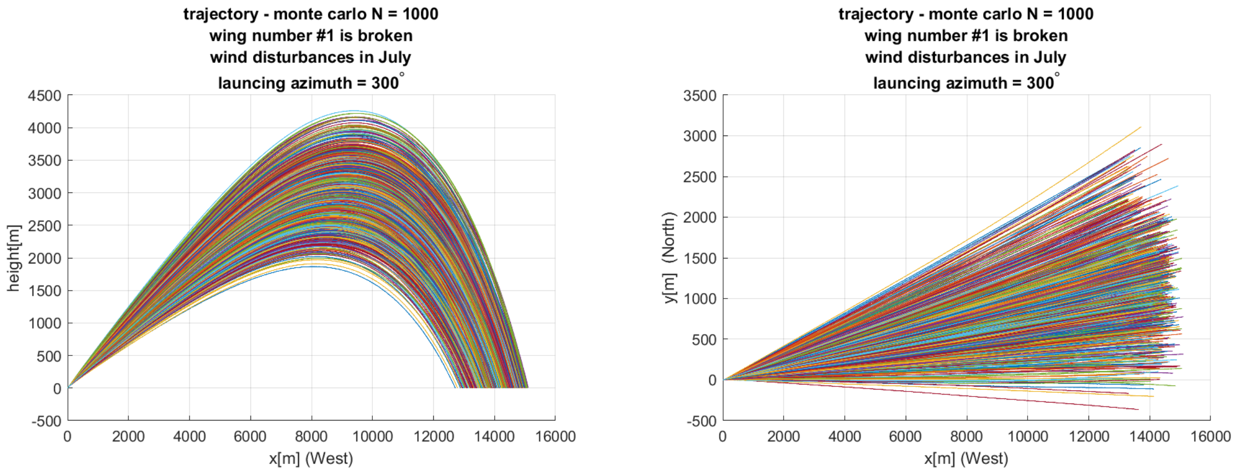
As could be expected, in the case of no broken wings, the rocket would not go back, and therefor we believe this case is safe.

Case of wing number 1 is broken

This case is shown in Figure 15.

Figure 15: wing number 1 is broken configuration

When running the *Monte Carlo* simulation the following results were obtained:



(a) height Vs x wing number 1 broken in july

(b) y Vs x wing number 1 broken in july

Figure 16: *Monte Carlo* trajectories for broken wing 1 in july

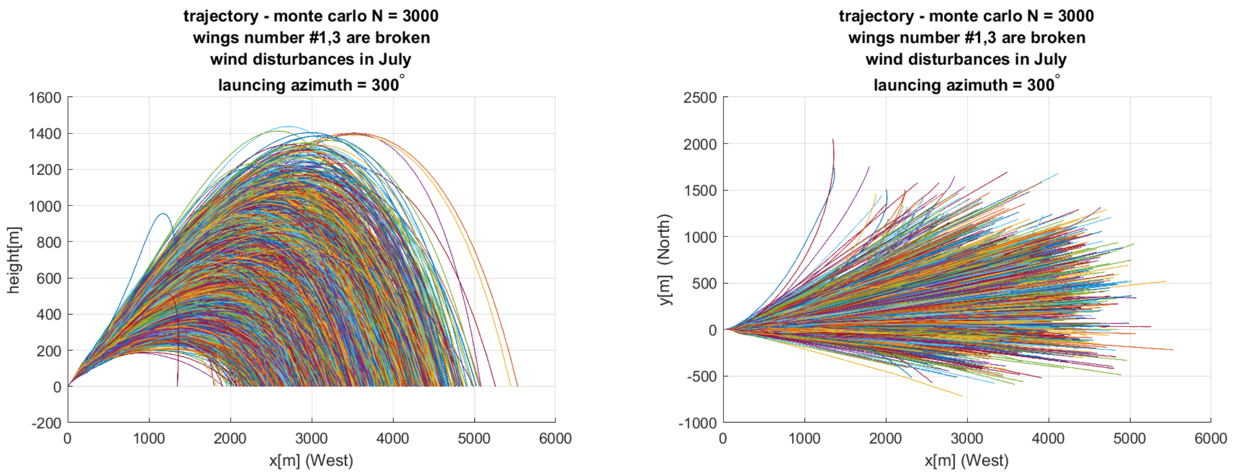
As can be seen, in the case of wing 1 broken, the rocket would not go back, and therefor we believe this case is safe. Moreover, in this configuration we see similar results to the configuration in which all the wings are not broken as seen in Figure 14.

Case of wings number 1,3 are broken

This case is shown in Figure 17.

Figure 17: wings number 1,3 are broken configuration

When running the *Monte Carlo* simulation the following results were obtained:



(a) height Vs x wings number 1,3 are broken in july

(b) y Vs x wings number 1,3 are broken in july

Figure 18: *Monte Carlo* trajectories for wings number 1,3 are broken in july

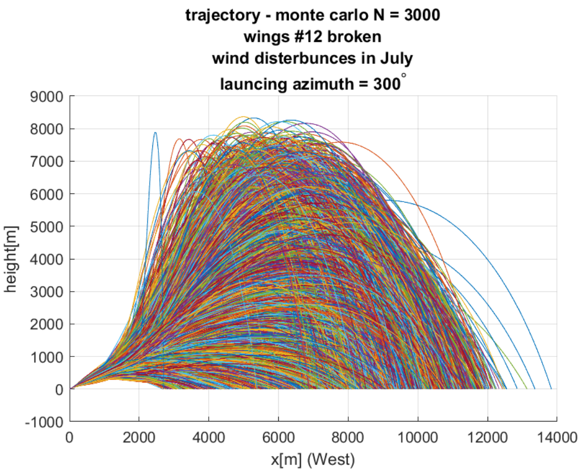
As can be seen, in the case of wing 1,3 broken, the rocket would not go back, and therefor we believe this case is safe.

Case of wings number 1,2 are broken

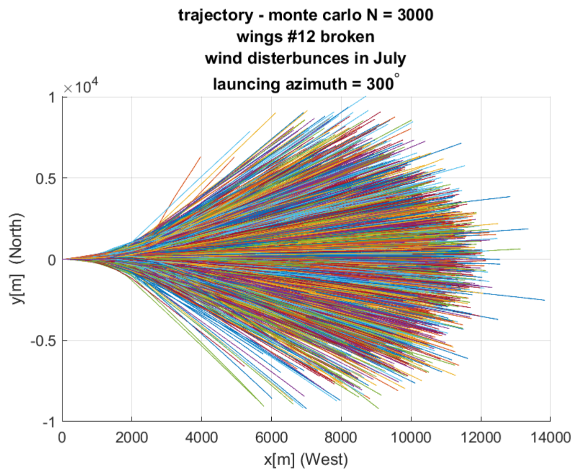
This case is shown in Figure 19.

Figure 19: wings 12 broken

When running the *Monte Carlo* simulation the following results were obtained:



(a) height Vs x wings number 1,2 are broken in july



(b) y Vs x wings number 1,2 are broken in july

Figure 20: *Monte Carlo* trajectories for wings number 1,2 are broken in july

As can be seen, in the case of wing 1,2 broken, the rocket would not go back, and therefor we believe this case is safe.

Conclusions

The validity of the modified 6DOF was tested by comparison with a 3DOF simulation and the original 6DOF simulation. In the two comparisons, we saw a strong correlation between the different results, which indicates the validity of our simulations.

In the following Table you could see the summery of all dangerous trajectories for all the possibilities of broken wings we have been asked to check:

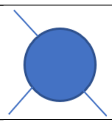
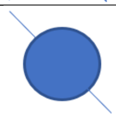
Configuration	number of dangerous trajectories (azimuth 300°)	number of numerical experiments
	0	3000
	0	1000
	0	3000
	0	3000

Table 3: summery of all dangerous trajectories

From the table above we can see that at azimuth 300° all the simulations gave us safe trajectories, and therefore we believe all of these cases are safe.