Cooperative Geometrical Rules and Guidance Laws for Simultaneous Target Interception

Research project

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Abstract

Guiding a team of cooperative interceptors for simultaneous interception has crucial effects, such as saturating the target countermeasures. The relationships between the interceptors in such a case can be classified into two kinds- coordination and cooperation. The first one means that the engagement is a collection of one-on-one scenarios unified by some design parameter, while the second means that information is continuously shared between the interceptors during the whole scenario. The merits of each class are discussed in this project via a literature review as well as implementations and simulations of existing geometrical rules and guidance laws.

Contents

1 Nomenclature

γ	=	path angle
ϵ	=	look angle
ϵ_{req}	=	required value of ϵ by the geometrical rule
$\Delta\epsilon$	=	the gap between ϵ to ϵ_{req}
λ	=	LOS angle
a	=	lateral acceleration of the missile
i	=	ith missile
K_P	=	gain for proportional controller
K_I	=	gain for integral controller
L	=	length of the instantaneous arc between the missile and the interceptor
M	=	missile mark
T	=	target mark
(X_M, Y_M)	=	missile position
(X_T, Y_T)	=	target position
Q_R	=	origin of the desired circular trajectory
R	=	radius of the desired circular trajectory
r	=	range between the target and the interceptor
t	=	time
t_d	=	desired impact time
t_f	=	final time
t_{go}	=	time-to-go
$\tilde{t}$	=	minimum possible impact time
u	=	control effort
V	=	speed of the missile
$X - O - Y$	=	inertial Cartesian reference frame
CIT	=	circular impact time
CE	=	control effort
LOS	=	line of sight
MAS	=	multi-agent system
PIP	=	predicted impact point
PI	=	proportional and integral controller
PN	=	proportional navigation

2 Introduction

Missile guidance refers to the process of designing the trajectory of an interceptor toward a predefined target, which is either stationary or moving. It is a hierarchical process that typically includes three levels: 1) geometrical rule, 2) guidance law, and 3) low-level control.

The top one, the geometrical rule, prescribes the desired geometrical path of the missile toward the target. In the case of a typical one-on-one engagement, there are traditionally two types of geometrical rules: two- and three-point. The first is based solely on the geometry between the interceptor and the target, while the second includes a reference point as well. A classic example of a two-point geometrical rule is parallel navigation. According to it, the direction of the line of sight (LOS) between the pursuer and the evader is kept parallel to the initial LOS. An example of a three-point geometrical rule is the LOS guidance. This rule requires that the missile will always be on the LOS between a reference point and the target. Using geometrical rules as the basis for the guidance design provides the reference course for the interceptor. Thus it is relatively simple to analyze the resulting trajectories and to implement the guidance loop.

The lower level in the guidance process is the guidance law. It is the algorithm that computes the maneuver command to enforce the desired geometrical rule. For instance, proportional navigation (PN) is the guidance law that implements the parallel navigation geometrical rule. The lateral acceleration is proportional to the LOS rate; thus, the trajectory is corrected until the LOS rate is nullified - this enforces the geometrical rule.

The bottom level in the guidance hierarchical process, which we will not focus on, is flight control and steering. At this level, the properties of the interceptor, as a rigid body, are taken into account. The goal of the flight control system is to compute the actuator commands for the steering mechanism subject to the guidance command [1].

Over the years, a great effort has been dedicated to the development of innovative geometrical rules and guidance laws that match the complexity of modern intercept scenarios. Their novelty stems from the possible intercept constraints, such as time- and geometrical constraints, and the use of an increased number of agents. Impact time constraint has a critical effect as it allows for salvo attacks to saturate target defense. Hence, this constraint encourages the cooperation of multiple pursuers for better performance. Therefore, cooperative guidance design based on simple geometrical principles, for a simultaneous interception, which requires a minimal amount of generally available information is of special interest. In missile guidance, cooperation can be obtained in two manners: implicit and explicit. In implicit cooperation, the scenario is broken down into individual one-on-one engagements that have a common design parameter, such as impact time. In explicit cooperation, the interceptor group acts as a unified team, in which the strategy of each interceptor continuously depends on the states of the other team-members. The implicit as well as the explicit method may yield improved results in different manners, as explained and demonstrated in the following project.

This project aims to investigate the implementation and performance differences between cooperation classes for achieving simultaneous interception. The project is organized as follows. First, the review of the following relevant topics is presented: 1) geometrical rules and guidance laws for imposing a desired impact time in one-on-one engagement, as the basis for implicit cooperation and 2) explicit cooperation for simultaneous interception. Next, fundamental definitions and the formulation of the basic guidance problem are introduced. Based on the previous sections, implementations and simulations of geometrical rules and guidance laws: one-on-one and cooperative, are presented and analyzed.

3 Literature Review

Salvo attack has crucial importance as saturating the target defense, limiting the target escape probability, and allowing reduced warhead. It can be implemented in two manners: coordination and cooperation.

3.1 Guidance with impact time constraint in one-on-one engagement

Coordination-based salvo attack typically means creating a collection of one-on-one engagements while imposing for each case the same desired impact time. Designing a salvo attack based on coordination enables a simple implementation. Additional merits and drawbacks for such a design stem from having a real-time connection to an external manager or not. Such connection enables better performances in the meaning of the resulting trajectories since the operator has a real-time overall view of the scenario. Yet, for such a connection, each interceptor should have a long-range communication system. In the presence of communication problems with this operator, what might occur for a long distance, such coordination loses its superiority. If the interceptors do not have a real-time connection with an external manager and are only defined to a desired impact time, no communication issues should be treated. However, given non-ideal conditions, such as drag or thrust effects, there might be differences between the real impact times, and therefore simultaneous interception will not occur [2]. In any case of coordination, the potential effect of cooperative interceptors for better performance, cannot be utilized.

The design of one-on-one guidance with an impact time constraint can be based on high-level design: development of geometrical rules or low-level design: development of guidance laws.

Using a high-level design, the Inscribed angle guidance, which is presented in [3] is developed. This geometrical rule is based on the fundamental geometrical principle which prescribes that inscribed angles in a circle, which subtend the same arc, are equal. This rule allows imposing a desired impact angle or time, by traversing a specific circular path, which is directly determined using the inscribed angle. An extension of the Inscribed angle guidance is proposed in [4]. This geometrical rule is developed based on the definition of an ellipse. According to it, the sum of the distances between each point along the ellipse and the two foci is constant. The Elliptic guidance enables enforcing impact angle and time and launch angle. This rule allows increasing the degrees of freedom to three, in comparison to [3]. This stems from the definition of an ellipse, depending on the origin as well as the rotation angle of the ellipse, and the distance between the foci. Additional geometrical rule is presented in [5]. This law is based on following a look-angle profile which is a polynomial in time. In [6], another geometrical rule which is also based on the look angle is introduced. It constrains the interceptor to follow a circular trajectory toward the target, in a required time. In contrast to the aforementioned rules, this rule enables imposing directly the impact time, without the need for any prior calculations, in a simple way.

Using low-level design, a guidance law is developed in [7]. The suggested guidance law is composed of two parts: the first one aims to reduce the miss distance, based on the PN guidance law, while the second aims to achieve the predefined impact time. For the implementation, feedback on the impact time error is added. This law is based on time-to-go estimation, which might be a source of error. As well as PN-based guidance laws for imposing a desired impact time, there are also existing guidance laws, which enforce the same constraint, based on nonlinear control theory. In [8] a Lyapunov- based guidance law is proposed. An accurate expression for the impact time is derived in terms of the initial heading error and is controlled by a single parameter. The articles mentioned above assumed the missile velocity to be constant. However, due to drag or thrust effects the velocity changes. To cope with this problem, an adaptive guidance scheme can be applied to provide robustness under varying conditions, as done in [9]. This scheme overcomes the varying velocity of a missile, by using the future average velocity.

Based on the aforementioned geometrical rules and guidance laws, by using geometrical rules as the basis for the whole guidance design, the resulting trajectories can be predicted, as well as the trends of some properties like the look angle. Furthermore, the design process is more simple and direct.

3.2 Cooperative guidance for simultaneous interception

Cooperation-based salvo attack typically refers to agents that aim to intercept the target simultaneously by an explicit dependence between them. The cooperative team is more robust in the meaning of having the ability to act independently, or via a minor dependence on an external operator. Such robustness increases the probability of the missiles intercepting the target simultaneously, even in the presence of unexpected events. However, the resulting trajectories might not be the ideal ones or easily predicted.

The following review is focused on the team dependence difference of a cooperative team: consensus-based approaches in a fully connected communication topology or a directed one.

An example of implementing a fully connected communication topology between the interceptors was proposed in [10]. The proposed cooperative proportional navigation guidance law has a time-varying navigation gain, which decreases the time-to-go variance cooperatively to achieve simultaneous interception. In [11], guidance laws based on the time-to-go error between all the missiles are presented. Based on two different estimations of time-to-go, two guidance laws were proposed. The maneuver command of both of them is divided into two components: normal for adjusting the curvature of the trajectory and tangent for adjusting the velocity magnitude. A two-staged guidance law was presented in [12]. The first one aimed to generate the desired initial conditions for the second stage- PN implementation. The acceleration command of each interceptor during the midcourse depends on the difference between its states to those of the other team members.

Implementing consensus protocols may also be on a directed communication topology. In multi-agent systems (MASs), there are typically two directed information exchange topologies: the cyclic and the leader-follower. The cyclic information exchange topology means that the i interceptor modifies its trajectory with respect to the $i+1$ interceptor. The merits of this framework are its simplicity and the minimum sensor information needed. In the leader-follower framework, the leader is usually independent of the other pursuers whereas they depend on the data received by the leader directly or in a chain fashion. Based on the cyclic framework, cooperative guidance law was proposed in [2]. Using sliding mode control, this law guarantees convergence to the desired impact time by achieving consensus in the time-to-go within a fixed time. In [13] cooperative geometrical rules and corresponding guidance laws to obtain a simultaneous interception are proposed. Simultaneous interception of a target at the minimum possible time can be achieved by using the cyclic strategy. It is proved that the impact time of each interceptor converged to the maximum impact time among the pursuers, in the case of heading straight to the target. Thus, max-consensus is achieved. Simultaneous interception of a target at any desired impact time can be achieved by using the leader-follower information exchange topology. The desired time is imposed by the leader and the followers modify their trajectories according to it. By using the leader-follower strategy, another cooperative guidance law was introduced in [14]. Based on feedback linearization, convergence of the impact time of the followers to that of the leader is obtained. Fully connected communication topology might provide better results, since each interceptor acts with respect to the whole engagement, while directed communication topology might be more practical for implementation.

Along with this review, the merits of the geometrical rule for the guidance design, as a direct way for design, which allows more efficient analyzing of the resulting trajectories, were emphasized. Moreover, the difference between coordination- a collection of one-on-one engagements and cooperation- a team of cooperative agents, as the basis for simultaneous interception by a team of interceptors, was introduced. The main trade-off is between better performance in the meaning of the resulting trajectories to robustness in the meaning of communication. Furthermore, the important effects of a salvo attack, such as saturating the target countermeasures were mentioned.

4 The guidance problem

4.1 Definitions

In this section, basic definitions from the guidance field are introduced, as a background for the formulation of the guidance problem in the next section.
The conventional one-on-one engagement scenario against a stationary target is demonstrated in Fig. 1.

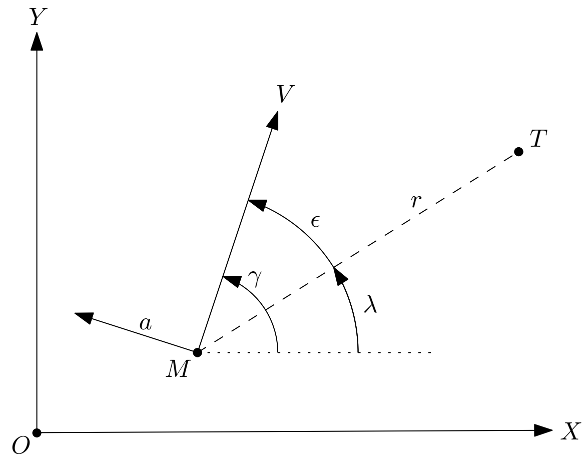


Figure 1: Planar one-on-one engagement geometry

The following notation is introduced

- M is the missile mark
- T is the target mark
- (X_M, Y_M) is the missile position
- (X_T, Y_T) is the target position
- r is the instantaneous range between the missile to the target
- V is the missile velocity
- a is the missile acceleration perpendicular to its velocity
- λ is the LOS angle, defined as the angle between the positive X axis to r
- γ is the missile path angle, which means the angle between the positive X axis to the missile velocity
- ϵ is the missile look angle, which is the angle between r to the missile velocity.

Based on these definitions, other terms were defined for a complete description of the guidance problem. The time that remains until the end of the engagement is denoted as t_{go} and is called time-to-go. It is formulated as:

$$t_{go} = t_f - t \tag{1}$$

where t_f is the desired time of flight and t is the time that elapsed from the beginning of the engagement. For performance evaluation, the control effort and the miss distance figures of merit are used. The control effort (CE) u is defined as:

$$u = \int_0^{tf} a^2 \, dt \tag{2}$$

Miss distance is the range at the time of the closest approach for zero interceptor control command. The geometrical rules designed against stationary targets can also be applied against the moving ones by using the predicted intercept point (PIP), which is the extrapolation of the known target trajectory until the specified impact time.

4.2 Formulation of the guidance problem

In this section, the equations of motion that usually describe the guidance problem are introduced, as a background for the implementation of geometrical rules in the next sections.
Based on elementary mechanics, the following equations are derived for the one-on-one case, against a stationary target, which is demonstrated in figure 1:

$$\begin{cases} \dot{\gamma} = \dfrac{a}{V} \\ \dot{X}_M = V \cos\gamma \\ \dot{Y}_M = V \sin\gamma \end{cases} \tag{3}$$

Based on the geometry, the next expressions can also be developed:

$$\begin{cases} r = (X_T - X_M)^2 + (Y_T - Y_M)^2 \\ \lambda = \arctan\left(\dfrac{Y_T - Y_M}{X_T - X_M}\right) \\ \epsilon = \gamma - \lambda \end{cases} \tag{4}$$

These equations are used to describe the kinematics at any moment. The results are compared to the required values by the relevant geometrical rule and the error is finally nullified by the guidance law. The objective of the guidance design is to find the control profile $a(t)$, which brings the missile in a sufficient vicinity of the target.

5 Simulations of geometrical rules for simultaneous interception

This section aims to present and simulate one-on-one and cooperative geometrical rules for impact time, based on the existing literature, for comparison between them in the next section.

5.1 Implicit cooperation for simultaneous interception

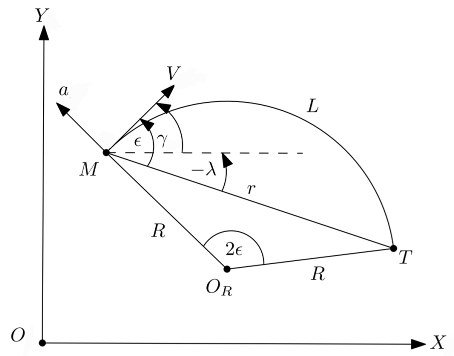


Figure 2: The planar engagement geometry of CIT (based on the illustration in [6])

The circular impact time (CIT) guidance, proposed in [6], is a one-on-one geometrical rule, against a stationary or constant-velocity moving target, for enforcing impact time, by traversing circular arcs. The planar engagement geometry between the target and the interceptor is shown in figure 2. Based on fundamental geometry principles:

$$\begin{cases} L = R \cdot 2\epsilon \\ R = \dfrac{r}{2 \cdot \sin\epsilon} \end{cases} \tag{5}$$

Where R is the radius of the desired circular path and L is the length of the arc between the interceptor and the target. Therefore,

$$L = \frac{r\epsilon}{\sin\epsilon} \tag{6}$$

Since, $\text{sinc}\epsilon = \sin\epsilon/\epsilon$, the formulation of the geometrical rule, if $V = const.$, is:

$$\epsilon_{req} = \text{Asinc}\left(\frac{r/t_{go}}{V}\right) \tag{7}$$

This geometrical rule is implemented by a simple PI controller:

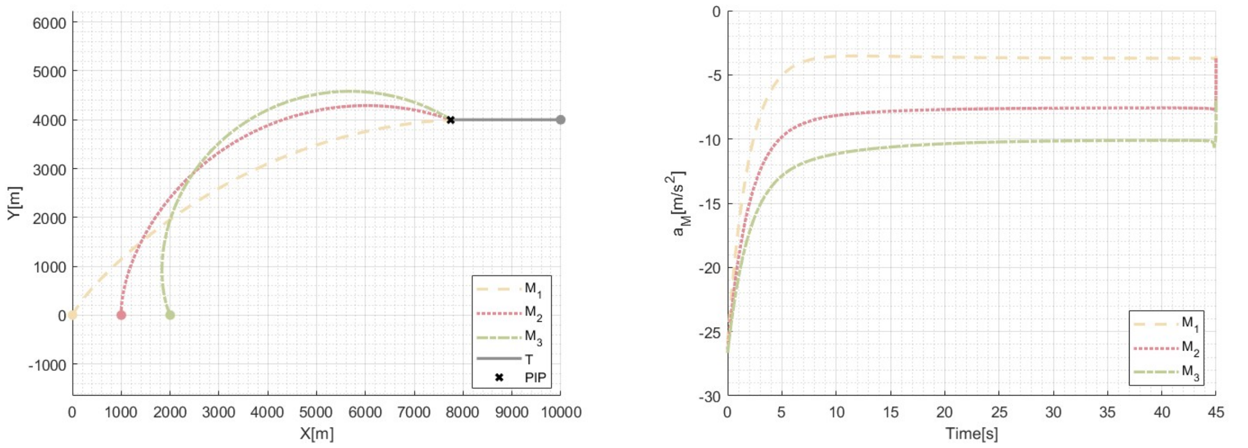
$$a = K_P \Delta\epsilon + K_I \int \Delta\epsilon \tag{8}$$

Where:

$$\Delta\epsilon = (\gamma - \lambda) - \epsilon_{req} \tag{9}$$

Designing a salvo attack based on this rule means that the cooperation between the interceptors is implicit since the geometrical rule is for a one-on-one case and the pursuers do not share any data between them.

The simulated scenario in the article of the proposed guidance law includes three interceptors, against a non-maneuvering target. The pursuers are launched from $(X_{M_1}, Y_{M_1}) = (0\ [\text{m}], 0\ [\text{m}]), (X_{M_2}, Y_{M_2}) =$



(a) Trajectories

(b) Acceleration

Figure 3: Implementation of [6], as implicit cooperation for impact time

$(1000\ [\text{m}], 0\ [\text{m}]), (X_{M_3}, Y_{M_3}) = (2000\ [\text{m}], 0\ [\text{m}])$, with speed of 200 [m/s] and path angles which are equal to $\gamma_1(0) = 62$ [deg] $, \gamma_2(0) = 92$ [deg] $, \gamma_3(0) = 113$ [deg]. The target is initially located at $(X_T, Y_T) = (10000\ [\text{m}], 4000\ [\text{m}])$, with speed and path angle of 50 [m/s] and 180 [deg] respectively. The desired impact time is 45 [s]. The gains of the controller are chosen to be $K_P = -150, K_I = -15$. The simulation is presented in figure 3.

5.2 Explicit cooperation for simultaneous interception

In [13] cooperative geometrical rules and corresponding guidance laws to obtain a simultaneous interception of a stationary target are proposed. The implementation is based on [6] and on the comparison between the interceptors of the minimum possible impact time, which is defined as:

$$\tilde{t}(t) = \frac{r}{V} \tag{10}$$

In this way, simultaneous interception of a target at the minimum possible time or at any desired impact time greater than the minimum is achieved. To ensure that, a simple PI controller, the same as introduced in 8, was proposed.

5.2.1 Cyclic strategy

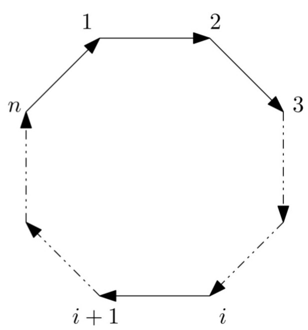
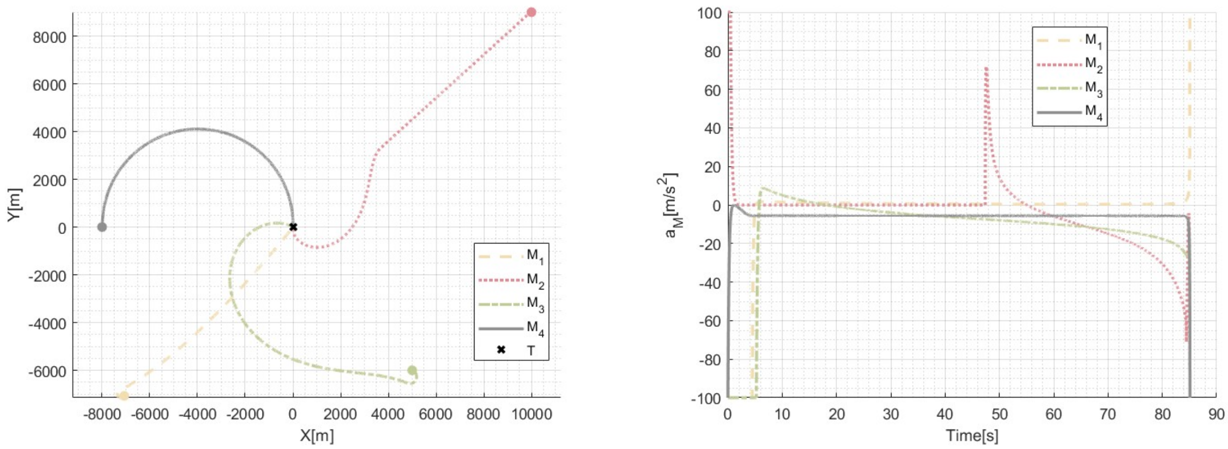


Figure 4: Cyclic communication topology (from [13])

Simultaneous interception of a target at the minimum possible time can be achieved by using the cyclic strategy which is demonstrated in figure 4. The geometrical rule is based on the principle that if one interceptor has a shorter minimum possible impact time, in comparison to its neighbor, it shapes its trajectory according to [6]. As a result, the trajectory is elongated and the impact time is increased, which allows the interceptor to synchronize its impact time with its neighbor. Otherwise, i.e. the interceptor has a longer minimum possible impact time, it nullifies its look angle. As a consequence, the interceptor heads straight to the target and reaches the target as fast as it can. By this geometrical rule, the impact time of each interceptor converged to the maximum $\tilde{t}$ among the pursuers, and hence, a max consensus is achieved. The mathematical formulation of the geometrical rule for minimum impact time is:

$$\epsilon_{req,i}(t) = \begin{cases} \text{Asinc}\left(\frac{\tilde{t}_i(t)}{\tilde{t}_{i+1}(t)}\right) & \tilde{t}_i(t) < \tilde{t}_{i+1}(t) \\ 0 & \tilde{t}_i(t) \geq \tilde{t}_{i+1}(t) \end{cases} \tag{11}$$

Where i represents the ith interceptor.
The engagement which is simulated in the article includes four pursuers against a stationary target. The pursuers are launched from $(X_{M_1}, Y_{M_1}) = (-7071\text{ [m]}, -7071\text{ [m]}), (X_{M_2}, Y_{M_2}) = (10000\text{ [m]}, 9000\text{ [m]}), (X_{M_3}, Y_{M_3}) = (5000\text{ [m]}, -6000\text{ [m]}), (X_{M_4}, Y_{M_4}) = (-8000\text{ [m]}, 0\text{ [m]})$, with speed of $V_1 = 125$ [m/s]$, V_2 = 180$ [m/s]$, V_3 = 170$ [m/s]$, V_4 = 150$ [m/s] and path angles which are equal to $\gamma_1(0) = 240$ [deg]$, \gamma_2(0) = 200$ [deg]$, \gamma_3(0) = 340$ [deg]$, \gamma_4(0) = 93$ [deg]. The target is located at $(X_T, Y_T) = (0\text{ [m]}, 0\text{ [m]})$. M_1 is chosen to be the leader. Hence, M_4 uses information from it, M_3 depends on M_4 and M_2 depends on M_3. The desired impact time is 70 [s]. For this simulation and for the next one, the gains of the controller are chosen to be $K_P = 800, K_I = 30$, and the acceleration is bounded to the maximum value of 100 [m/s^2]. The simulation is presented in figure 5.



(a) Trajectories (b) Acceleration

Figure 5: Implementation of [13], as explicit cooperation for minimum impact time

5.2.2 Leader-follower strategy

Simultaneous interception of a target at a desired impact time can be achieved by using the leader-follower communication topology. One of the interceptors is arbitrarily chosen to be the leader. The leader implements the CIT geometrical rule with the desired impact time, without any dependence

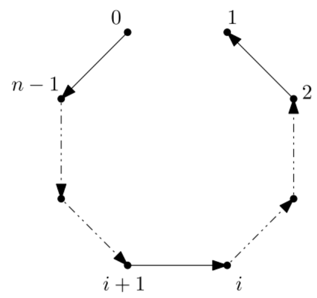


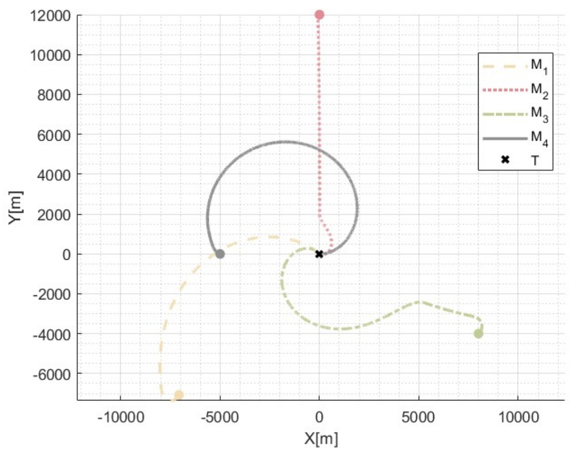
Figure 6: Leader-follower communication topology: one-to-one (from [13])

on the other interceptors. The other interceptors are the followers. One type of leader-follower communication topology is one-to-one. According to it, the ith follower adjusts its trajectory according to the $i+1$th follower, as demonstrated in figure 6. In fact, the desired impact time is imposed by the leader and the followers modify their trajectories according to it. Thus, simultaneous interception at any desired impact time is achieved. The geometrical rule is formulated as follows:

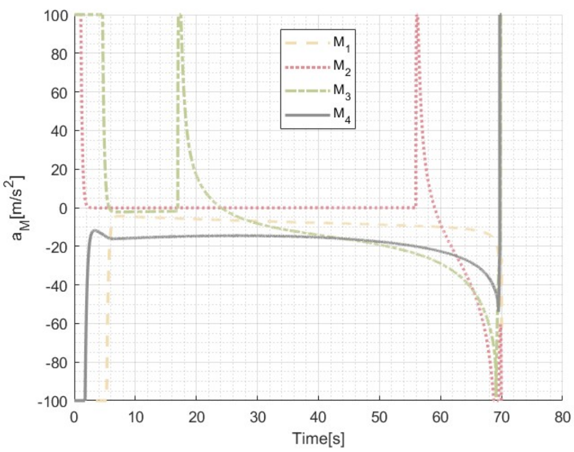
$$\begin{aligned} \epsilon_0(t) &= \text{Asinc}\left(\frac{\tilde{t}_0(t)}{t_d - t}\right) \\ \epsilon_i(t) &= \begin{cases} \text{Asinc}\left(\frac{\tilde{t}_i(t)}{\tilde{t}_{i+1}(t)}\right) & \tilde{t}_i(t) < \tilde{t}_{i+1}(t) \\ 0 & \tilde{t}_i(t) \geq \tilde{t}_{i+1}(t) \end{cases} \end{aligned} \tag{12}$$

When the subscript 0 represents the leader, the subscript $i = 1, 2, ..., n-1$ represents the ith follower and t_d is the desired impact time.

The engagement that is simulated in the article includes four pursuers against a stationary target. The pursuers are launched from $(X_{M_1}, Y_{M_1}) = (-7071 \text{ [m]}, -7071 \text{ [m]}), (X_{M_2}, Y_{M_2}) = (0 \text{ [m]}, 12000 \text{ [m]}), (X_{M_3}, Y_{M_3}) = (8000 \text{ [m]}, -4000 \text{ [m]}), (X_{M_4}, Y_{M_4}) = (-5000 \text{ [m]}, 0 \text{ [m]})$, with speed of $V_1 = 210$ [m/s]$, V_2 = 180$ [m/s]$, V_3 = 220$ [m/s]$, V_4 = 240$ [m/s] and path angles which are equal to $\gamma_1(0) = 250$ [deg]$, \gamma_2(0) = 230$ [deg]$, \gamma_3(0) = 40$ [deg]$, \gamma_4(0) = 170$ [deg]. The target is located at $(X_T, Y_T) = (0 \text{ [m]}, 0 \text{ [m]})$. M_1 is chosen to be the leader. Hence, M_4 uses information from it, M_3 depends on M_4 and M_2 depends on M_3. The desired impact time is 70 [s]. The simulation is presented in figure 7.



(a) Trajectories



(b) Acceleration

Figure 7: Implementation of [13], as explicit cooperation for impact time

6 Simulations for comparison between implicit- and explicit- cooperation

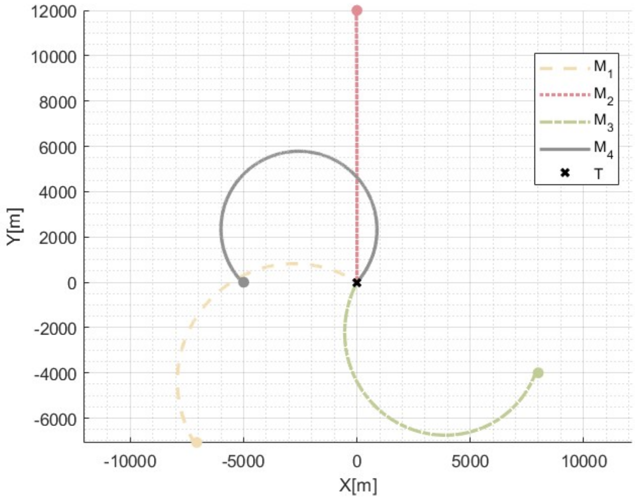
This section aims to examine the performance differences between implicit- and explicit- cooperation, and even between different communication topologies, via simulations, based on implementations of the geometrical rules mentioned above.

6.1 Minimum impact time simulation

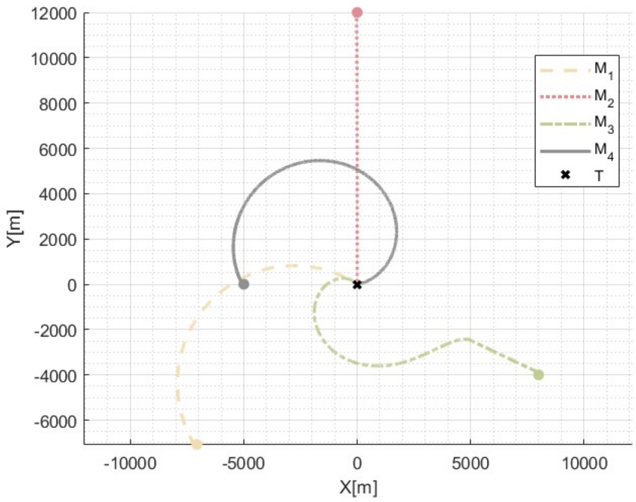
Based on the aforementioned geometrical rules, the engagement of guiding a cooperative team for a simultaneous interception at the minimum possible impact time can be designed in three manners: coordination, cyclic, and leader-follower communication topologies between the interceptors. Hence, for comparing the performance differences between these classes, simulations of this scenario are presented next.

6.1.1 Simulation

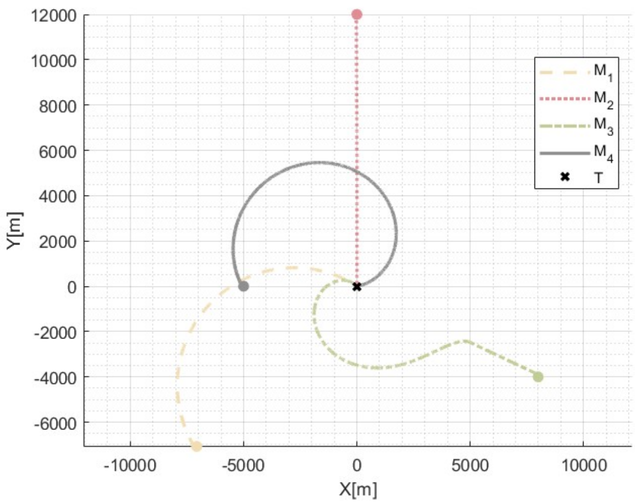
The engagement which is simulated in this section includes four interceptors against a stationary target. The pursuers are launched from $(X_{M_1}, Y_{M_1}) = (-7071 \text{ [m]}, -7071 \text{ [m]}), (X_{M_2}, Y_{M_2}) = (0 \text{ [m]}, 12000 \text{ [m]}), (X_{M_3}, Y_{M_3}) = (8000 \text{ [m]}, -4000 \text{ [m]}), (X_{M_4}, Y_{M_4}) = (-5000 \text{ [m]}, 0 \text{ [m]})$, with speed of $V_1 = 210$ [m/s], $V_2 = 180$ [m/s], $V_3 = 220$ [m/s], $V_4 = 240$ [m/s] and path angles which are equal to $\gamma_1(0) = 250$ [deg], $\gamma_2(0) = 230$ [deg], $\gamma_3(0) = 40$ [deg], $\gamma_4(0) = 170$ [deg]. The target is located at $(X_T, Y_T) = [0 \text{ [m]}, 0 \text{ [m]}]$. By the cyclic topology, the interceptors converged to intercept the target at the minimum possible impact time by definition. For the CIT and the leader-follower topology, the desired impact time is set to the minimum possible impact time, which is equal to $\max[\tilde{t}_i(0)] = \frac{r_2(0)}{V_2} = 66.6$ [s]. The gains of the controller are chosen to be $|K_P| = 800, |K_I| = 30$. The simulations are presented in figure 8. The max acceleration and the control effort, for each missile during the engagement, are presented in tables 1-4.



(a) Implicit cooperation based on CIT



(b) Explicit cooperation based on cyclic topology



(c) Explicit cooperation based on leader-follower topology

Figure 8: Minimum impact time simulation: trajectories comparison

Strategy	Max acceleration [m/s^2]	Control effort [m^2/s^3]
One-on-one strategy	1764	423967
Leader follower strategy	1764	423964
Cyclic strategy	1764	430533

Table 1: Minimum impact time simulation: performances comparison of M_1

Strategy	Max acceleration [m/s^2]	Control effort [m^2/s^3]
One-on-one strategy	559	35503
Leader follower strategy	559	35504
Cyclic strategy	712	38167

Table 2: Minimum impact time simulation: performances comparison of M_2

Strategy	Max acceleration [m/s^2]	Control effort [m^2/s^3]
One-on-one strategy	2893	1170303
Leader follower strategy	1584	394837
Cyclic strategy	2434	423559

Table 3: Minimum impact time simulation: performances comparison of M_3

Strategy	Max acceleration [m/s^2]	Control effort [m^2/s^3]
One-on-one strategy	4331	62555
Leader follower strategy	1226	110999
Cyclic strategy	5086	589898

Table 4: Minimum impact time simulation: performances comparison of M_4

6.1.2 Discussion

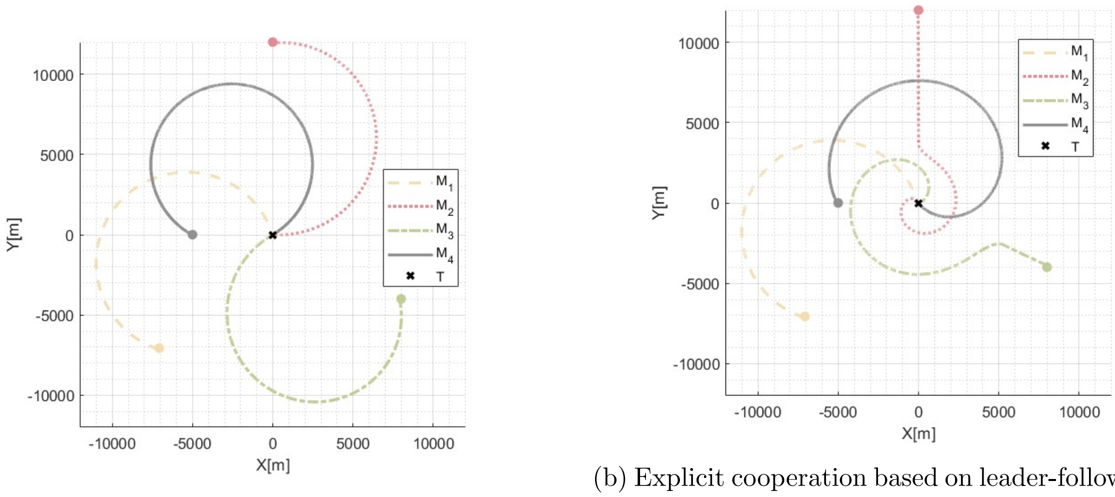
The main difference between the cyclic strategy to the other methods: the CIT and the leader-follower, is the dependence on an external operator. The cyclic method does not enable an external constraint to be enforced. Therefore, the cyclic method converges by itself to the minimum possible impact time- $t_f = 66.7$ [s]. This impact time is not equal to the calculated minimum impact time, which is set for the other methods- 66.6 [s] due to heading errors. Beyond the gap in the impact time result, there is a trend regarding the CE: for most of the missiles, a greater CE is required in the case of cyclic strategy. This stems from the convergence process in the cyclic topology, which demands continuous computations along the scenario- at any moment, none of the team members has a whole predefined trajectory, and the next step is continuously updated. Only in the case of M_3, the CE is higher in the case of CIT collection, due to heading errors, relative to the resulting trajectories by each method. It emphasizes that better performances can also be obtained for the cooperation case.

6.2 Desired impact time simulation

Based on the geometrical rules mentioned above, the performances of coordination and leader-follower topology between the interceptors can also be compared via simulations for simultaneous interception at a desired impact time, greater than the minimum one.

6.2.1 Simulation

The engagement which is simulated in this section is the same as that simulated in the previous section, but, here, the desired impact time is set to 110 [s]. The simulations are presented in figure 9. The max acceleration and the control effort, for each missile, are presented in tables 5-8.



(a) Implicit cooperation based on CIT

(b) Explicit cooperation based on leader-follower topology

Figure 9: Desired impact time simulation: trajectories comparison

Strategy	Max acceleration [m/s^2]	Control effort [m^2/s^3]
One-on-one strategy	1249	207033
Leader follower strategy	1248	207058

Table 5: Desired impact time simulation: performances comparison of M_1

Strategy	Max acceleration [m/s^2]	Control effort [m^2/s^3]
One-on-one strategy	1875	401739
Leader follower strategy	3462	91507

Table 6: Desired impact time simulation: performances comparison of M_2

Strategy	Max acceleration [m/s^2]	Control effort [m^2/s^3]
One-on-one strategy	3340	1558197
Leader follower strategy	1584	401008

Table 7: Desired impact time simulation: performances comparison of M_3

Strategy	Max acceleration [m/s^2]	Control effort [m^2/s^3]
One-on-one strategy	4585	27389
Leader follower strategy	1225	109587

Table 8: Desired impact time simulation: performances comparison of M_4

6.2.2 Discussion

The main difference between the CIT collection to the leader-follower topology is how much the dependence on an external operator exists. For the CIT, each missile should have an external constraint to be enforced. However, for the leader-follower strategy, only the leader is set to a desired intercept time and the other team members converge to it. As a consequence, a longer computational effort is required which leads to higher CE for two missiles. For the other missiles, the CE is higher for the case of CIT collection due to larger heading errors relative to the resulting trajectories.

7 Conclusions

Guiding a team of missiles to simultaneous target interception has crucial merits such as saturating the target defense and, hence, increasing the probability of mission success. Such guidance can be designed in two manners: coordination and cooperation.

Coordination-based design may usually provide better performance in terms of the resulting trajectories and control effort, due to an external supervisor, who has an overall view of the engagement. Hence, this supervisor can efficiently plan the trajectories of each missile to get the desired cooperative attack. Therefore, the path of the interceptors is known and there is no need for a complicated computational process. However, if there is a real-time connection with that supervisor, the superiority mentioned above might be lost in the presence of communication problems, which may occur in the case of long-range missiles. If there is no real-time connection with this external manager, the robustness of the interceptors, in terms of overcoming unexpected changes along the engagement, is lost.

Cooperation-based approach provides more robustness in terms of communication, since not all, and maybe, none of the interceptors is dependent on an external operator. Such an approach can be implemented via two communication topologies: the leader-follower and the cyclic. The leader-follower, in the one-on-one meaning, allows enforcing an external constraint, by the leader. This is translated into less CE requirements due to the needless of a complex convergence process. However, the method is less robust, due to the critical dependence on the leader. The cyclic communication topology is robust, due to the independence of the system. Yet, this method requires a significant CE most of the time.

The communication method among the interceptors needs to be chosen according to the specific engagement requirements and properties. The importance of the impact time constraint for a team of interceptors, and the simplicity stems from high-level design, leads to potential efficient guidance concepts. Therefore, there is a need for investigating and developing cooperative geometrical rules for simultaneous interception. Designing such rules, even for ideal cases, has critical importance as a basis for potentially more realistic future development.

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