Starting Vortices
1st Reseatch Project

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Chapter 1

Introduction

The study of unsteady flow around airfoils has been a subject of ongoing research in the field of aerospace engineering[1]. Understanding the dynamics of the flow and its effects on the circulation of the near field is crucial for the design and optimization of flying bodies such as aircraft. This paper aims to investigate the unsteady flow around a flat airfoil, with a focus on the impact of heaving or accelerating flow on turbulence evolution and performance analysis.

Previous studies have shown that unsteady flow around airfoils can have significant effects on the performance of flying bodies[2]. By analyzing the effect of heaving or accelerating flow on the circulation of the near field, we aim to gain a deeper understanding of the turbulence evolution and how it affects the performance of flying bodies. This research is essential for those in the aerospace industry who strive to improve the efficiency and stability of aircraft.

We will also investigate the evolution of the wake behind the airfoil over time. By understanding the dynamics of the wake, we can aim to design aircraft that are more efficient with less drag and lift force fluctuations. Flapping and unsteady flows are studied to get an understanding for how the near field changes at these conditions. Using thin-airfoil theory and Kelvin's theorem, we will analyze the formation of turbulence around an airfoil and how it impacts the behavior of the object in flight.

The wakes and turbulence generated behind a wing in flight have long been a subject of intense study. The violation of the steady state of flow can lead to the formation of vortices at the trailing edge of the wing. This paper will focus specifically on two methods of changing the induced angle of attack, Heaving the wing up and down and accelerating the fluid flow, which changes the relative velocity and therefore the magnitude of the angle of attack. We will investigate the effects of these methods on the circulation of the near field and the formation of turbulence

When a body moves through a fluid, wakes and turbulence can form at the trailing edge. The transition from a steady state to an unsteady state causes vortices to form, which can be either attached or detached. Attached vortices rotate with the body, while detached vortices do not. Heaving, or oscillation about an equilibrium position, can also occur as a result of the wakes and turbulence. The amplitude and frequency of the heaving motion can affect the magnitude of the pressure differential and circulation of the detached vortices.

Unsteady flow conditions can cause significant changes in the aerodynamic forces experienced by an aircraft. These conditions can be caused by wakes from other vehicles, turbulence in the atmosphere, or vortices that form around the body of the aircraft. These time-varying forces can cause the aircraft to enter an unsteady state, which can be mitigated through the use of trailing edge devices such as flaps or slats. These devices can help to reduce the heaving motion of the aircraft and keep it closer to its trimmed flight condition. Additionally, active control systems can be used to adjust these devices in response to changing conditions, allowing for better management of unexpected flow conditions, ensuring safety and efficiency.

Chapter 2

The Problem

The problem description chapter presents the mathematical and geometrical background for the study on starting vortices in continuum flow. It provides an overview of the relevant concepts and equations, highlighting the key features and characteristics of these vortices. This chapter also summarizes the current state of knowledge in the field, discussing the open questions and challenges that motivate the study. The overall aim of the study is then outlined, along with the specific research questions that will be addressed in the subsequent chapters.

2.1 General Description

Let C be a closed loop enclosing the same fluid elements and located in a 2D flow field. The time-dependent velocity $U(t)$ is measured with degree of angle of attack α. α may be the real angle of attack, or the effective angle of attack – induced by the heaving motion of the airfoil (cyclic vertical movement of the airfoil, in a direction perpendicular to itself). The circulation around the airfoil at time t expressed as

$$\Gamma(t) = \oint U(t) \cdot d\ell. \tag{2.1}$$

If we consider the flow to be ideal and assuming there is no flow besides that already present, Kelvin's theorem tells us[3][4] that the material derivative ($\frac{D}{Dt}$) is zero for a fluid flow with no external forces

$$\frac{D\Gamma}{Dt} = 0. \tag{2.2}$$

That is, the circulation around the airfoil remains constant.

When a flow velocity changes, Kelvin's theorem shows that vortices are allowed to form. These vortices can only evolve on the trailing edge. In a discrete vortex model of the airfoil wake, we release a concentrated vortex each time step. The size of this is fixed by the Kelvin theorem and can be controlled with the instantaneous changes in airflow. We will simulate the location of the vortices in the wake over time.

As can be seen in Figure 2.1, the airfoil was placed along the abscissa of the complex plane where the leading edge is right on the origin. Complex notation will be used to better simplify combined flow fields. To model vortex formation over time, we will impose an iterative calculation. For example, on the kth iteration, when it's said that the time is equal to $t = t_k$ and the nascent

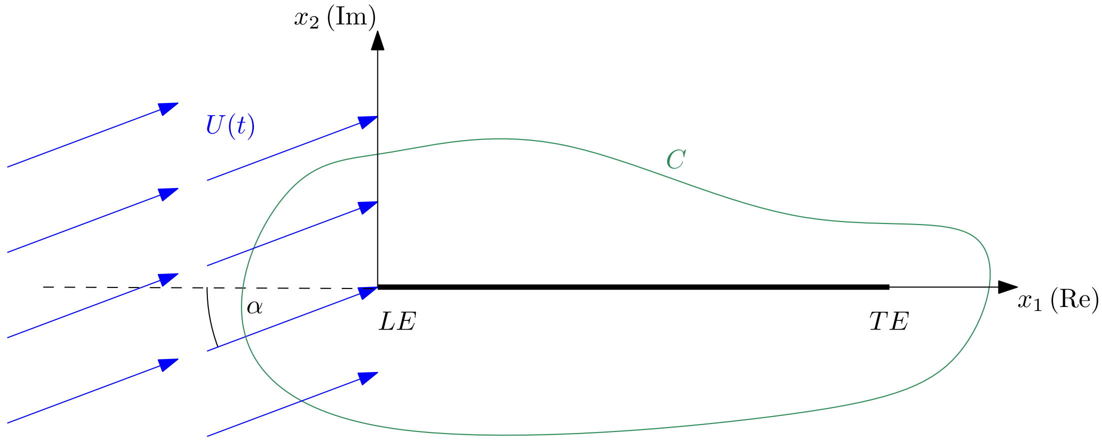

Figure 2.1: Conceptual representation of the problem. The angle of attack, denoted by α, can be either the explicit angle of attack or the induced angle of attack.

vortex marked as Γ_k (at this time, $k-1$ vortices have already formed), complex notation should show (all of the variables are dimensionless)

$$W(z) = U(t)e^{i\alpha} - \frac{i}{2\pi}\left(\sum_{k=1}^{N}\frac{\Gamma_k}{z - z_{\Gamma_k}} + \frac{\Gamma_c}{z - z_c}\right). \tag{2.3}$$

The airfoil is described as a surface of infinitesimal vortices with the function $\gamma(x_1, t)$ – function of the location along the real axis, and time. c is the length of the chord line – the distance between x_1^{LE} and x_1^{TE}. Therefore, the whole circulation around the airfoil is simply given by

$$\Gamma_c(t) = \int_0^c \gamma(x_1, t)\, dx_1 \qquad , \qquad \begin{aligned} z_{\mathrm{LE}} &= 0 + 0i \\ z_{\mathrm{TE}} &= c + 0i \end{aligned}. \tag{2.4}$$

In order to solve Γ_k, we must satisfy the impermeability condition – fluid flow perpendicular to the airfoil should be zero at any time. Applying the thin airfoil theory with the right complex notation combined with impermeability condition (using equations 2.3, 2.4) yields

$$\begin{aligned} \operatorname{Im}\left(\underset{0<z<c}{W(z)}\right) &= 0 \\ &= \operatorname{Im}\left(U(t)e^{i\alpha} - \frac{i}{2\pi}\left(\sum_{k=1}^{N}\frac{\Gamma_k}{z - z_{\Gamma_k}} + \fint_0^c \frac{\gamma(x_1, t)}{z - x_1}\, dx_1\right)\right). \end{aligned} \tag{2.5}$$

2.2 Analytical and Numerical Analysis

Starting at t_0, the circulation around the airfoil is set to be Γ_0 and there is no vortices shed by the airfoil at all. From this time and on, following Kelvin's theorem, the whole circulation around the airfoil is conserved as Γ_0 and therefore the magnitude of the nascent vortex at time t_N calculated by

$$\Gamma_N = -\left(\sum_{n=1}^{N-1} \Gamma_n + \int_0^c \gamma(x_1,t)\, dx_1 - \Gamma_0\right). \tag{2.6}$$

Using a common change of variables for the infinitesimal circulation function, $\gamma(x_1,t)$ as

$$x_1 = \frac{c}{2}\left(1-\cos(\theta)\right),$$

then expanding $\gamma(\theta,t)$ to series of sin functions (who identically satisfying Kutta[5] condition $\gamma(\pi,t)=0$) and using Glauert's equation[6], yields

$$\fint_0^\pi \frac{\gamma(\theta,t)}{\cos(\theta)-\cos(\theta_0)} \sin(\theta)\, d\theta = 2\pi U(t)\left[A_0 - \sum_{n=1}^{\infty} A_n \cos(n\theta_0)\right] \tag{2.7}$$

and the calculation of A_i is done by placing 2.7 in 2.5. Applying these on L more distinct locations along the airfoil, arises the matrix problem

$$\mathbb{M}\mathbf{A} = \alpha \mathbf{I} - U^{-1}\,\mathrm{Im}\left(\frac{i}{2\pi}\mathbf{\Gamma}\right) \tag{2.8}$$

where

$$\mathbb{M} = \begin{pmatrix} 1-\frac{1}{2}\mathrm{Im}\left(\frac{i}{z_0-z_{\Gamma_N}}\right) & -\frac{2}{\pi}\mathrm{Im}\left(\frac{i}{1\left(z_0-z_{\Gamma_N}\right)}\right)-\cos(\theta_0) & -\cos(2\theta_0) & \cdots & -\frac{2}{\pi}\mathrm{Im}\left(\frac{i}{L\left(z_0-z_{\Gamma_N}\right)}\right)-\cos(L\theta_0) \\ 1-\frac{1}{2}\mathrm{Im}\left(\frac{i}{z_1-z_{\Gamma_N}}\right) & -\frac{2}{\pi}\mathrm{Im}\left(\frac{i}{1\left(z_1-z_{\Gamma_N}\right)}\right)-\cos(\theta_1) & -\cos(2\theta_1) & \cdots & -\frac{2}{\pi}\mathrm{Im}\left(\frac{i}{L\left(z_1-z_{\Gamma_N}\right)}\right)-\cos(L\theta_1) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1-\frac{1}{2}\mathrm{Im}\left(\frac{i}{z_L-z_{\Gamma_N}}\right) & -\frac{2}{\pi}\mathrm{Im}\left(\frac{i}{1\left(z_L-z_{\Gamma_N}\right)}\right)-\cos(\theta_L) & -\cos(2\theta_L) & \cdots & -\frac{2}{\pi}\mathrm{Im}\left(\frac{i}{L\left(z_0-z_{\Gamma_N}\right)}\right)-\cos(L\theta_L) \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} A_0 & A_1 & \ldots & A_L \end{pmatrix}^T$$

$$\mathbf{\Gamma} = \begin{pmatrix} \sum_{k=1}^{N-1}\left(\frac{\Gamma_k}{z_0-z_{\Gamma_k}} - \frac{\Gamma_k}{z_0-z_{\Gamma_N}}\right) + \frac{\Gamma_0}{z_0-z_{\Gamma_N}} \\ \sum_{k=1}^{N-1}\left(\frac{\Gamma_k}{z_1-z_{\Gamma_k}} - \frac{\Gamma_k}{z_1-z_{\Gamma_N}}\right) + \frac{\Gamma_0}{z_1-z_{\Gamma_N}} \\ \vdots \\ \sum_{k=1}^{N-1}\left(\frac{\Gamma_k}{z_L-z_{\Gamma_k}} - \frac{\Gamma_k}{z_L-z_{\Gamma_N}}\right) + \frac{\Gamma_0}{z_L-z_{\Gamma_N}} \end{pmatrix}$$

This vortex initially positioned very close to the Trailing edge

$$z_{\Gamma_k}\big|_{t_k} = c + \varepsilon$$

From this moment and on, on each time step t_k, the flow field is calculated at each vortex position z_{Γ_k} by

$$W_{\Gamma_k}(z) = Ue^{i\alpha} - \frac{i}{2\pi}\left(\sum_{\substack{m=1\\ m\neq k}}^{N} \frac{\Gamma_m}{z_{\Gamma_k}-z_{\Gamma_m}} + \int_0^c \frac{\gamma(x_1,t)}{z_{\Gamma_k}-x_1} dx_1\right) \tag{2.9}$$

And then the position of Γ_k in time step t_n set by

$$z_{\Gamma_k}\big|_{t_n} = z_{\Gamma_k}\big|_{t_{n-1}} + dt \cdot W\left(z_{\Gamma_k}\right)\big|_{t_{n-1}}.$$

Chapter 3

Results

The results chapter presents the findings of the study on the formation of starting vortices in a continuum flow. The simulations were designed to investigate the effects of various factors on the formation and evolution of these vortices, without any focus on their impact on the overall aerodynamic performance of the system. This chapter presents the results of these simulations, highlighting the key findings and their implications for the understanding and control of starting vortices in continuum flow.

3.1 Acceleration

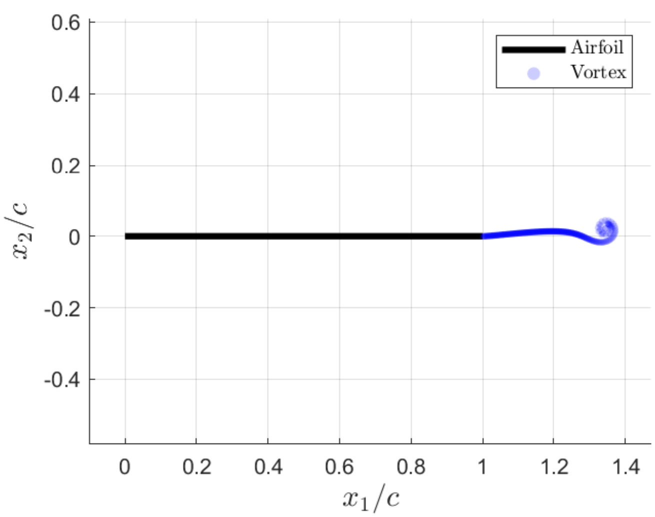

Figure 3.1: Vortices position on the imaginary plain for certain time frame – Linear acceleration perturbation of the steady state

In a simulation of a stationary airfoil with its leading edge set at the origin of the plane and pointing in the direction of the real axis, it was observed that the presence of the airfoil resulted in the formation of vortices as the fluid moved around it. These vortices, also known as wake vortices, were visible in the simulation as swirling patterns of fluid that trailed behind the airfoil. The formation of these vortices is a common phenomenon that occurs when a fluid moves around an object, and it is a result of the fluid's movement being disrupted by the presence of the airfoil. The strength and shape of the vortices can vary depending on a number of factors, including the shape of the airfoil and the properties of the fluid. In the simulation, the vortices were observed to be relatively strong and well-defined, indicating that the fluid was moving at a significant speed and disrupting the flow around the airfoil in a significant way.

On Figure 3.1 the fluid accelerated linearly from $U(t)/U_0 = 1$ to $U(t)/U_0 = 1.3$ with constant angle of attack of 3°. At each time step (overall 300 time steps), a vortex forms at the rear tip of the airfoil and then moves on the plane according to the flow regime dictated by the turbulence caused by the presence of the airfoil itself, as well as previously formed vortices. The simulation results indicate the presence of a vortical flow pattern in the form of a curly chain of vortices. These vortices are observed to drift in the direction of the fluid flow and curl around the very beginning of the chain (the right hand end). The chain ends (chronological) at the airfoil's trailing edge. This behaviour of starting vortices is familiar and correspondent to some experiments made[7] and other researches[1][2][8].

3.2 Heaving

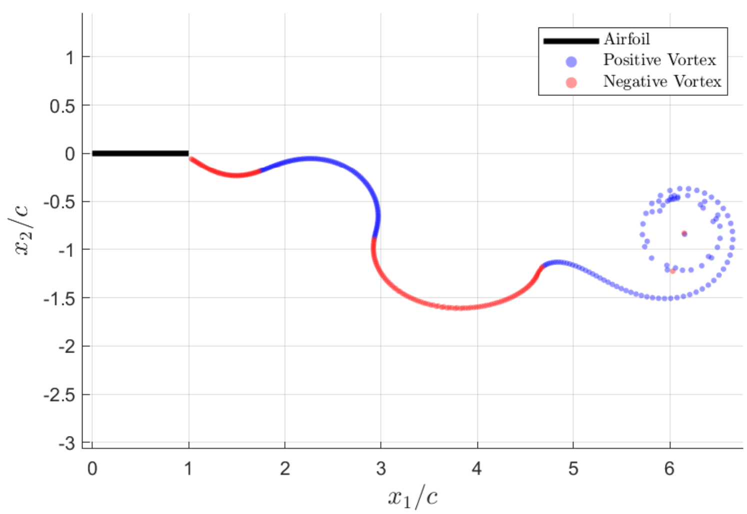

Figure 3.2: Vortices position on the imaginary plain for certain time frame – Symmetric Heaving perturbation of the steady state

During a simulation of a moving airfoil following a symmetric heaving pattern, it was observed that the airfoil caused the formation of vortices in the fluid as it moved past the airfoil while the airfoil was oriented towards the real axis. The vortices, which were visible in the simulation as swirling patterns of fluid with changing direction that trailed behind the airfoil, likely matched the frequency of the heaving motion. The formation of vortices, which is a common occurrence when a fluid moves around an object, is a result of the fluid's movement being disrupted by the presence of the airfoil. The strength and shape of the vortices can vary based on various factors. It is clear that when the airfoil moved along the positive direction of the imaginary axis, the vortices formed with a positive trend (opposite to the direction of the clock) and when it moved in the opposite direction, the vortices also had an opposite trend. The vortices in the simulation were observed to be strong and well-defined, which suggests that the fluid was moving at a high enough speed and significantly disrupting the flow around the airfoil.

In Figure 3.2 the fluid is held at a constant velocity of $U(t)/U_0 = 1$ and the airfoil generates an effective angle of attack due to its movement. The airfoil has a sinusoidal pattern, where $\psi(t) = 0 + \epsilon \sin(2\pi f t) \cdot i$ represents the position of the airfoil at time t. for $\epsilon \ll 1$ the airfoil is assumed to be located on the real axis, that is $\psi(t) \approx 0 + 0i$ with changing velocity $\dot{\psi}(t)$. The parameters that were used during the simulation at the source of Figure 3.2 were (non dimensional)

$$\begin{aligned} \epsilon &= 5 \cdot 10^{-2} \\ f &= 1 \end{aligned} \tag{3.1}$$

At each time step (overall 350 time steps), a vortex is created at the projection of the back end of the airfoil on the real axis, and then it moves on the plane based on the flow conditions caused by the turbulence generated by the airfoil and any previously formed vortices. Simulation results show the presence of a vortical flow pattern in the form of a chain of vortices arranged in a curved, cyclical shape. These vortices are observed to drift in the opposite direction to the initial movement of the airfoil and curl around the very beginning of the chain (the right hand end).

Chapter 4

Conclusion

The conclusion chapter of this study on unsteady flow around a flat airfoil summarizes the key findings and provides insight into their significance. The study investigated the formation and evolution of vortices in continuum flow, with a focus on understanding the effects of heaving and accelerating flow on the circulation of the near field. The results of the simulations demonstrate the presence of a vortical flow pattern in the form of a chain of vortices arranged in a curved, cyclical shape.

The implications of these findings are numerous. They provide a deeper understanding of the behavior of vortices in unsteady flow conditions, which can be used to improve the performance and efficiency of aircraft. The results also suggest new directions for future research, such as investigating the use of trailing edge devices or active control systems to manage the effects of unsteady flow conditions on aircraft. Overall, this study advances our understanding of unsteady flow around a flat airfoil and provides a foundation for future research in this area.

The results of the simulation with the linear accelerating fluid flow were particularly noteworthy. The simulation showed the formation of vortices at the rear tip of the airfoil, which then moved according to the flow regime caused by turbulence and previously formed vortices. The simulation results indicate the presence of a vortical flow pattern in the form of a curly chain of vortices, which are observed to drift in the direction of the fluid flow and curl around the very beginning of the chain. These results were consistent with previous experiments and studies, adding to the body of knowledge on the behavior of vortices in unsteady flow conditions. Furthermore, the results of this simulation provide a better understanding of how the circulation changes over time and what happens with turbulence evolution, which is crucial for performance analysis.

The results of the simulation with the heaving motion of the airfoil were informative as well. The simulation showed the formation of vortices at the projection of the back end of the airfoil on the real axis. The results indicate the presence of a vortical flow pattern in the form of a chain of vortices arranged in a curved, cyclical shape. These vortices were observed to move based on the flow conditions caused by the turbulence generated by the airfoil and any previously formed vortices. This behavior of starting vortices is consistent with previous experiments and studies, adding to the body of knowledge on the behavior of vortices in unsteady flow conditions. The results of this simulation provide a better understanding of how the circulation changes over time and what happens with turbulence evolution, which is crucial for performance analysis.

The research conducted in this project has revealed that aerodynamic starting vortices are a highly intricate and constantly changing phenomenon. These vortices have the potential to significantly affect the performance of aircraft. Through the course of this research, several key findings were made regarding the characteristics and behavior of these vortices. It was found that the strength and persistence of the vortices are influenced by a range of factors, including the velocity of the fluid flow, the effective angle of attack and the amplitude and the frequency of the heaving. In addition, the vortices were found to interact with other flow structures in the surrounding air, leading to complex two-dimensional flow patterns. These findings have important implications for the design and operation of aircraft, as well as for the development of advanced control and sensing systems.

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