Geostationary Satellite Stationkeeping Using Milankovitch-Lyapunov

Research Project

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1 Introduction

The history of geostationary satellite stationkeeping originates in the early days of satellite communications, following the launch of the first geostationary satellite, Syncom 3, in 1964.

Stationkeeping involves maintaining a satellite's position within its designated orbital slot, a process that has became crucial as more satellites were lunched into the geostationary Earth orbits (GEO) to facilitate global communications. Consequently, GEO satellites stationkeeping is widely-studied. Some significant advancements in that domain were made by analyzing the GEO resonances affecting longitudinal motion [1] and the associated east-west impulsive stationkeeping maneuvers [2], followed by the development of autonomous low-thrust stationkeeping algorithms in the early 1980s [3]. A discussion of the various aspects of impulsive and low-thrust GEO stationkeeping can be found in Ref. [4].

Recently, electric propulsion systems (EP) are gradually replacing the chemical propulsion systems as a means for GEO satellite stationkeeping. According to Ref. [5], all-electric GEO satellites, are associated with a significant total mass reduction due to a decrease of the required propellant, as well as with a concomitant reduction in launcher cost, while gaining a considerably extended total lifespan for the GEO satellites, compared to chemical or hybrid variants.

However, electric propulsion systems introduce several operational challenges [6]. Modulating the thrust magnitude in EP systems by adjusting the power or by a pulse-width modulation scheme is possible, but may result in reduced efficiency. Consequently, EP systems are often operated at a constant thrust level. As a consequence, thrusters adhere to an on-off discrete firing profile. Additional control constraints in EP systems include the requirement for a minimum time interval between consecutive thruster firings, allowing the system to dissipate heat and stabilize the power supply, as noted by Ref. [7].

Recent advancements in GEO stationkeeping using EP include implementing numerical approaches such as differential inclusion [8] and convex optimization [9] to satisfy mission constraints, while using non-singular orbital elements as the state variables [9,10]. Ref. [11] also discussed an optimization-based maneuver planner for stationkeeping of GEO satellites using electric propulsion, while Ref. [12] proposed using deep neural-networks for stationkeeping, addressing the highly-nonlinear nature of the involved dynamics.

In thous studies, the prevalent approach was using an orbital elements-based dynamical model and a set of constraints, to obtain a GEO stationkeeping scheme. The orbital elements are estimated using ground measurements [13]. However, one of the most rapidly-advancing technologies to obtain state estimates of GEO satellites is utilizing Global Navigation Satellite Systems (GNSS) [14]. Recent successful implementations of GNSS receivers onboard GEO satellites include the GOES-R satellite, utilizing the Viceroy-4 GPS receiver for onboard orbit determination with meter-level accuracy [15]. Moreover, recent research suggests that integrating GPS with other navigation systems such as BeiDou can further improve accuracy [16].

GNSS provide direct measurements of the position and velocity vectors in an

Earth-centered, Earth-fixed (ECEF) frame. Therefore, it is natural to develop a stationkeeping method that utilizes these measurements, and provides a closed-loop control scheme which promotes the autonomy of GEO satellites. Recognizing this trend, Ref. [17] developed a nonlinear closed-loop control law that utilized direct ECEF position and velocity measurements, while satisfying the longitude and latitude tolerance constraints, and requiring a reasonable velocity change, but did not include a non-linear stability proof for the developed feedback control law, showing only local linear stability results.

One possibility for developing closed-loop stationkeeping for GEO satellites utilizing the GNSS-measured ECEF position and velocity, is to transform the position and velocity vectors into the Milankovitch vectorial elements, i.e. the orbital angular momentum vector and eccentricity vector, which are simple functions of the position and velocity.

The Milankovitch vectorial elements have long been recognized as advantageous in terms of numerical implementations, offering an elegant, simple and geometrically-insightful representation of orbital perturbation theory [18, 19]. The Milankovitch elements were successfully applied to low-thrust Earth-orbit trajectory optimization using a concise and non-singular representation of the perturbed orbital model [20].

The purpose of this research project is to build on the results of Ref. [21] that in developed a closed-loop stationkeeping control law for GEO satellites based on the Milankovitch vectorial elements, and extend the results by optimizing fuel consumption and allowing for multiple-satellite collocation.

Ref. [21] provides a nonlinear stability proof based on Lyapunov's second method. Thus, Ref. [21] extends previous results by developing a control algorithm with proven global asymptotic stability at any desired station. The work gives an explicit analytical closed-form expressions for the control acceleration components in the ECEF frame. Furthermore, as another extension of existing literature, it is proven that the newly-developed stationkeeping control law is invariant under averaged J_2 perturbations, and is robust to thrust misalignment and magnitude errors.

Simulations in Ref. [21] show that the newly-developed stationkeeping method requires only 10 mN of thrust to satisfy the standard longitude and latitude stationkeeping tolerance constraints under perturbations that include Earth's triaxiality, lunisolar gravitational perturbations, and solar radiation pressure. Taking to account practical aspects of EP system, the work presents continuous, discrete, and constant-magnitude-thrust implementations of the stationkeeping control law. The simulations also indicate that the total annual ΔV for the discrete version of the stationkeeping control law is smaller than the one reported in Ref. [17].

Furthermore, if the thrust level is increased to 25 mN, the newly-developed closed-loop stationkeeping exhibits a significant reduction in the duty cycle of the EP system, while maintaining a reduction in the annual ΔV.

2 Control Law

The control law designed in Ref. [21] is given by

$$\mathbf{u} = -[k_h \mathbf{\Delta h^T} \tilde{\mathbf{r}} + \frac{k_e}{\mu} \mathbf{e^T} (\tilde{\mathbf{v}} \tilde{\mathbf{r}} - \tilde{\mathbf{h}})]^T \tag{1}$$

where $\mathbf{e} = [e_x, e_y, e_z]^T$ is the eccentricity vector, $\mathbf{h} = [h_x, h_y, h_z]^T$ is the angular momentum vector, $\mathbf{r} = [x, y, z]^T$, $\mathbf{v} = [x' - \omega y, y' + \omega x, z']^T$ are the position and velocity vectors of the satellite in the ECEF system, μ is the gravity constant of Earth, and k_h, k_e are the control gains. In addition,

$$\Delta \mathbf{h} = \mathbf{h} - \mathbf{h_d} \tag{2}$$

where $\mathbf{h_d}$ is the desired angular momentum and is determined by

$$\mathbf{h_d} = [0, 0, \hat{R_d}^2 \omega]^T = [0, 0, (R + k_\lambda(\lambda - \lambda_d - \lambda_b))^2 \omega]^T \tag{3}$$

$$\hat{R_d} = R + k_\lambda(\lambda - \lambda_d - \lambda_b) \tag{4}$$

where R is the geostationary radius, k_λ is the longitude control gain, λ is the satellite real longitude, λ_d is the desired sub-satellite longitude, and λ_b is a station-dependant bias.

Note that the notation $\tilde{x}$ for vector $\mathbf{x}$ denotes the cross-product matrix equivalent. The closed-form expression of the control law acceleration in the ECEF system can be written as

$$u_x = \frac{k_e}{\mu}[((\omega x + y')y + z'z)e_x + (\omega y^2 - x'y + h_Z)e_y + (\omega yz - x'z - h_z)e_z]$$

$$-k_h[(\hat{R}_d^{\,2}\omega - h_z)y + zh_y] \tag{5}$$

$$u_y = \frac{k_e}{\mu}[(-\omega x^2 - xy' - h_z)e_x + ((-\omega y + x')x + z'z)e_y + (-\omega xz - y'z + h_x)e_z]$$

$$+k_h[(\hat{R}_d^{\,2}\omega - h_z)x + zh_x] \tag{6}$$

$$u_z = \frac{k_e}{\mu}[(-xz' + h_y)e_x + (-yz' - h_x)e_y + (x'x + y'y)e_z] - k_h(h_x y - h_y x) \tag{7}$$

3 Preilimanry Results

During this research project, a simulation was conducted to check the control law for Continuous thrust of 10 mN. The simulation was conducted for a satellite similar in parameters to the Eutelsat 115 West B satellite with a mass of 2205 kg and a sun-projected cross-sectional area of 51.7 m^2. The satellite desired longitude was $\lambda_d = -10deg$ and the initial conditions for the simulation was a deviation of 2 km in each ECEF axis from the desired station.

For this simulation we chose the control parameters and longitude bias as detailed in Table 1.

Under these parameters we obtained steady-state tolerances of ± 0.01 deg in longitude and ± 0.03 deg in latitude, as shown in Fig.1. The overall calculated velocity change for one year was $\Delta V = 68.8\,\frac{m}{sec}$.

$k_e[\frac{km^2}{sec^3}]$	$k_h[\frac{1}{km^2 sec}]$	$k_\lambda[\frac{km}{rad}]$	$\lambda_b[deg]$
2.97×10^{-5}	1.3×10^{-15}	3200	-0.056

Table 1: simulation Parameters

Figure 2 shows the the thrust acceleration components and the total thrust acceleration as a function of time for one year. In the plot of the total acceleration, the maximum acceleration capability of the thruster is also shown. the maximum total thrust acceleration during one year of simulation is $u_{\max} = 4.427 \times 10^{-9} \, \frac{km}{sec^2}$.

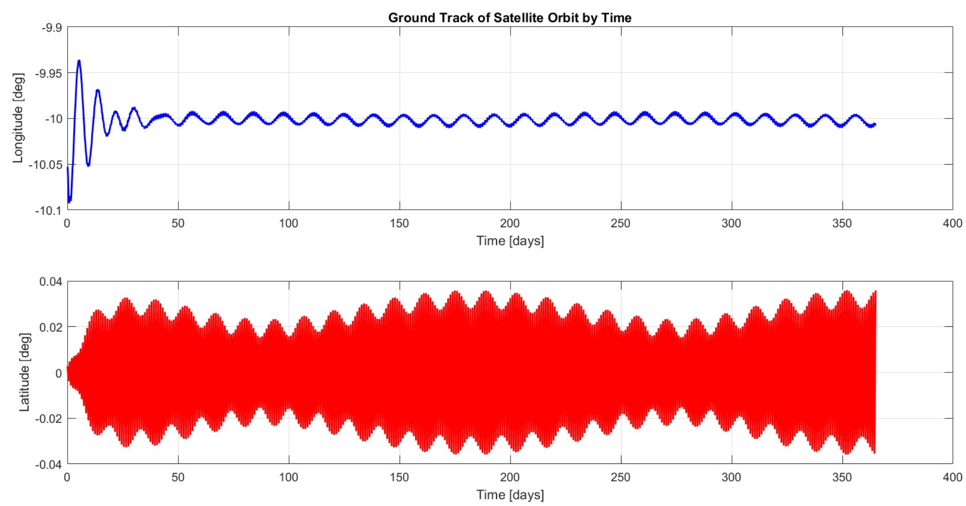

Figure 1: Time history of longitude and latitude

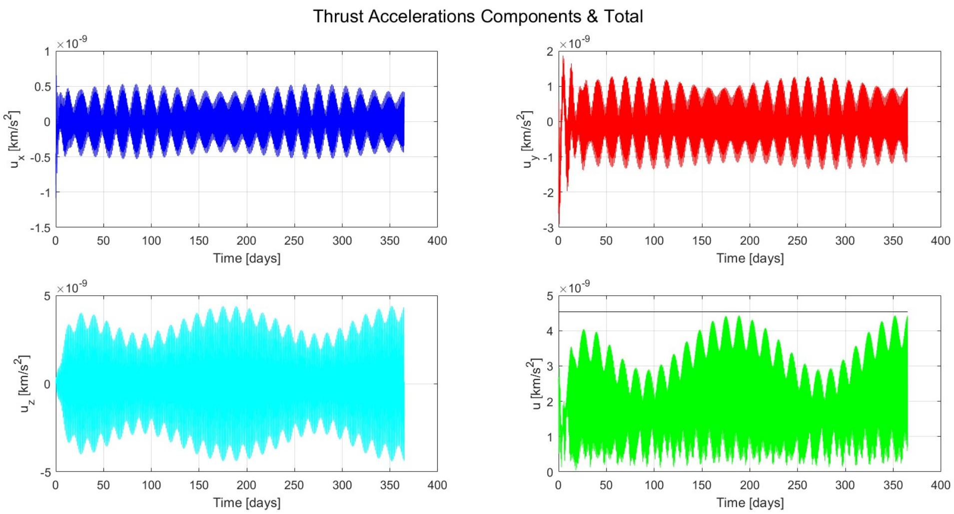

Figure 2: Time history of thrust acceleration components and total thrust acceleration

4 Future Research

In future research we will look into the following ideas:

- Optimization of control gains: We will try to find the optimized control parameters (k_h, k_e, k_λ) for minimum annual ΔV.
- Introducing constraints inside the control law, by using Lyapunov Barrier Functions [22]: We would like to consider the longitude and latitude constraints in the control design stage using Lyapunov Barrier Functions, instead of manually adjusting the control parameters.
- To adapt the control law for multiple-satellite collocation: We would like to use the control law designed to find the maximum number of satellites that can be collocated in a certain slot using the control laws developed.

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