Geostationary Satellite Gain Optimization for Closed-Loop Stationkeeping

Research Project

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March 17, 2025

1 Introduction

Control laws often require gain tuning. In Ref. [1], a new stationkeeping control law for geostationary satellites was designed, based on eccentricity and angular momentum matching. This control law includes control gains and control parameters that can be manipulated in order to minimize fuel consumption while adhering to the latitude and longitude constraints.

In this research paper, the control law from Ref. [1] is adapted for continuous, discrete, and constant magnitude thrust, and the control parameters are separated into groups. Optimal control gains are identified for each thrusting method, demonstrating a reduction in ΔV compared to previous studies, while ensuring the satellite is still within the desired area in space. The discrete thrust method achieves the best performance, with a ΔV of 55.5 m/s/year.

2 Control Parameters

The control law designed in Ref. [1] and reconstructed in my first research paper [2] is given by

$$\mathbf{u} = -[k_h \mathbf{\Delta h^T} \tilde{\mathbf{r}} + \frac{k_e}{\mu} \mathbf{e^T} \left(\tilde{\mathbf{v}} \tilde{\mathbf{r}} - \tilde{\mathbf{h}} \right)]^T \tag{1}$$

where $\mathbf{e} = [e_x, e_y, e_z]^T$ is the eccentricity vector, $\mathbf{h} = [h_x, h_y, h_z]^T$ is the angular momentum vector, $\mathbf{r} = [x, y, z]^T$, $\mathbf{v} = [x' - \omega y, y' + \omega x, z']^T$ are the position and velocity vectors of the satellite in the ECEF system, μ is the gravity constant of Earth, and k_h, k_e are the control gains. In addition,

$$\Delta \mathbf{h} = \mathbf{h} - \mathbf{h_d} \tag{2}$$

where $\mathbf{h_d}$ is the desired angular momentum and is determined by

$$\mathbf{h_d} = [0, 0, \hat{R_d}^2 \omega]^T = [0, 0, (R + k_\lambda (\lambda - \lambda_d - \lambda_b))^2 \omega]^T \tag{3}$$

$$\hat{R_d} = R + k_\lambda (\lambda - \lambda_d - \lambda_b) \tag{4}$$

where $R = 42164.16$ km is the geostationary radius, k_λ is the longitude control gain, λ is the satellite real longitude, λ_d is the desired sub-satellite longitude, and λ_b is a station-dependent bias.

There are four control gains and parameters that can be change in order to find optimized values under constrains – $k_h, k_e, k_\lambda, \lambda_b$, where we can split them into two groups – long timescale (k_h, k_e) and short timescale (k_λ, λ_b). This paper will focus on the long-timescale parameters; therefore, we will set k_λ, λ_b to be the same as in the last study, while our optimized variables are k_h, k_e.

In addition, the control law was modified for three different thrust implementations: Continuous thrust, subject to the maximum thrust acceleration u_0,

$$\mathbf{u}_C = \begin{cases} \mathbf{u} & \|\mathbf{u}\| \leq u_0 \\ \mathbf{0} & \|\mathbf{u}\| > u_0 \end{cases} \tag{5}$$

Discrete thrust, subject to chosen acceleration thresholds $(\sigma_{x,y,z})$, as described in Ref. [3],

$$(u_{x,y,z})_D = \begin{cases} \sigma_{x,y,z}\text{sign}(u_{x,y,z}) & |u_{x,y,z}| \geq \sigma_{x,y,z} \\ 0 & |u_{x,y,z}| < \sigma_{x,y,z} \end{cases} \tag{6}$$

where $\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} = u_0$, and constant–magnitude thrust,

$$\mathbf{u}_M = \begin{cases} u_0 \dfrac{\mathbf{u}}{\|\mathbf{u}\|} & \|\mathbf{u}\| \geq u_0 \\ 0 & \|\mathbf{u}\| < u_0 \end{cases} \tag{7}$$

The maximal thrust acceleration is $u_0 = 4.5351 \times 10^{-9}$ km/s^2 which corresponds to 10 mN thrust of thrust for a 2205-kg satellite.

Note that for the discrete thrust method, we have two additional control parameters, because we can choose the ratio between the three thresholds; so, for example, we can choose σ_x, σ_z as optimization parameters while σ_y will be determined by the maximal acceleration u_0.

In the case of constant-magnitude thrust, the thruster can be separated from the satellite by using mechanical gimbals. Thus, the thruster can be directed to any direction, obtaining thrust-steering independently of the satellite body orientation [4].

3 Optimization

In order to find the optimal k_h, k_e for the control law, we define the following optimization problem.

We wish to minimize the total velocity change during one year,

$$\min_{k_h,k_e}(\Delta V) = \min \int_0^{t_f} \|\mathbf{u}\| dt \tag{8}$$

The operational constraints are defined in terms of latitude (δ) and longitude (λ) limits. Therefore, the optimization problem becomes

$$\min_{k_h,k_e}(\Delta V) = \min \int_0^{t_f} \|\mathbf{u}\| dt \tag{9}$$

$$s.t. \begin{cases} \delta^2 - \Delta_s^2 & \leq 0 \\ (\lambda - \lambda_d)^2 - \Delta_s^2 & \leq 0 \end{cases} \tag{10}$$

where $\lambda_d = 10°, \Delta_s = 0.05°$.

3.1 Optimal Continuous Thrust

For the optimization of the control law using continuous thrust the optimization problem is

$$\min_{k_h,k_e}(\Delta V) = \min \int_0^{t_f} \|\mathbf{u}_C\| dt \tag{11}$$

$$s.t. \begin{cases} \delta^2 - \Delta_s^2 & \leq 0 \\ (\lambda - \lambda_d)^2 - \Delta_s^2 & \leq 0 \end{cases} \tag{12}$$

In this research, the other control gains were kept as in Ref. [1], namely $k_\lambda = 3200\text{km}^2/\text{rad}, \lambda_b = -0.056°$, and the initial conditions for the simulation were a deviation of 2 km in each ECEF axis from the desired station, and the correct velocity of the desired orbit.

Using MATLAB's `fmincon` function, we were able to search for the optimal set of control gains $[k_h, k_e]$ and found that for $k_h = 9.3926 \times 10^{-16} 1/(\text{km}^2\,\text{s}), k_e = 5.48482 \times 10^{-6}\text{km}^2/\text{s}^3$ the cost is $\Delta V = 62.795$ m/s/year, thus saving 6.8 m/s/year compared to the results in Ref. [1]. While maintaining the satellite with the safety area in space, as shown in Fig. 1, with $\Delta\lambda = \pm 0.0083°, \Delta\delta = \pm 0.047°$, which is less less stringent than in Ref. [1]. Using these gains the acceleration did not reach saturation at any point of time, as shown in Fig. 2.

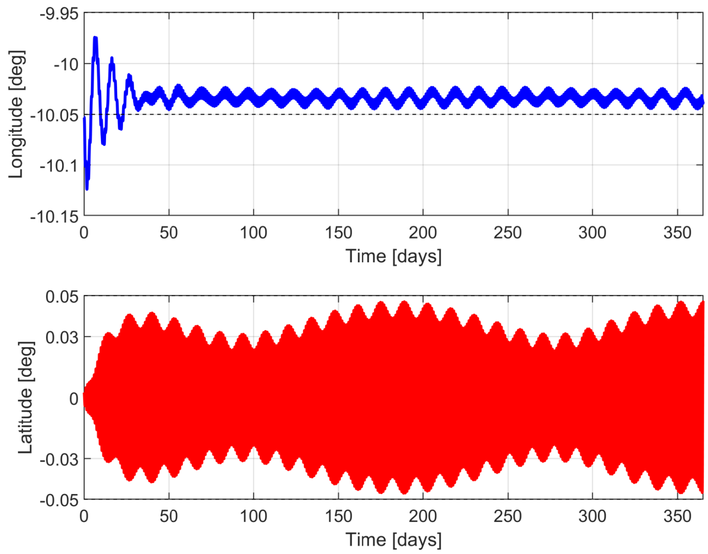

Figure 1: Ground track For optimal continuous thrust

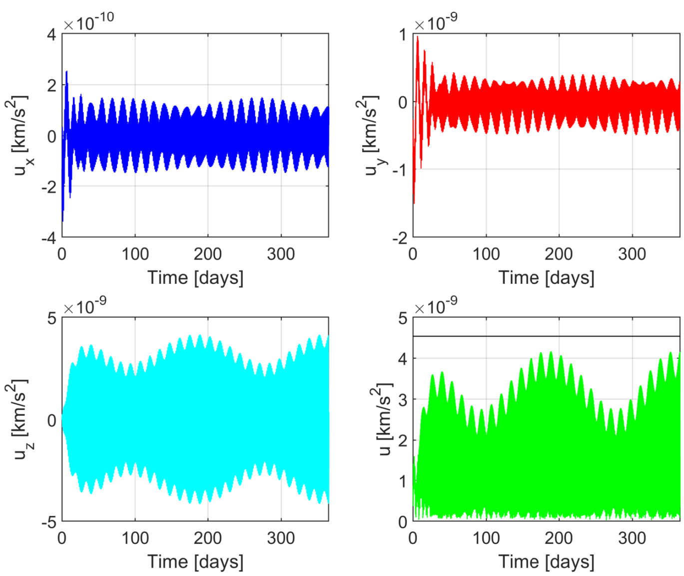

Figure 2: Thrust acceleration components and total acceleration for continuous thrust

3.2 Optimal Discrete Thrust

For the optimization of the control law using discrete thrust our optimization problem will be:

$$\min_{k_h,k_e}(\Delta V) = \min \int_0^{t_f} \|\mathbf{u}_D\| dt \tag{13}$$

$$s.t. \begin{cases} \delta^2 - \Delta_s^2 & \leq 0 \\ (\lambda - \lambda_d)^2 - \Delta_s^2 & \leq 0 \end{cases} \tag{14}$$

In this research, the other control gains kept as in Ref. [1], namely $k_\lambda = 3200\text{km}^2/\text{rad}, \lambda_b = -0.022°$, and the initial conditions for the simulation were a deviation of 2 km in each ECEF axis from the desired station, and the correct velocity of the desired orbit. In addition, the thresholds were taken as in Ref. [1] for 10mN thrusters: $\sigma_x = 6.62 \times 10^{-10}\text{km/s}^2, \sigma_y = 1.1 \times 10^{-9}\text{km/s}^2, \sigma_z = 4.14 \times 10^{-9}\text{km/s}^2$.
Using MATLAB's fmincon function, we were able to search for the ideal set of control gains $[k_h, k_e]$ and found that for $k_h = 1.6407 \times 10^{-15}\ 1/(\text{km}^2\,\text{s}), k_e = 6.8936 \times 10^{-6}\,\text{km}^2/\text{s}^3$ the cost is $\Delta V = 55.55$ m/s/year, thus saving 7.7 m/s/year compared to Ref. [1] results for the same case.

While maintaining the satellite with the safety area in space, as shown in Fig. 3, with $\Delta\lambda = \pm 0.0176°, \Delta\delta = \pm 0.0436°$, which is less stringent on the latitude and longitude compare to the results of Ref. [1] for the same case.

Using these gains, there was no use of the x-axis component of the control acceleration, as shown in Fig. 4, similar to Ref. [1] results. In addition we found the duty cycle of the acceleration in y-axis to be 11.5% and in z-axis to be 46.6% with a repetitive pattern as can be seen in Fig. 5.

Another attempt was done to try and find an optimal combination of long-timescale gains and thrust thresholds. The results got the thresholds used in Ref. [1]; the optimal combination that was found is $\sigma_x = 6.62 \times 10^{-10}\,\text{km/s}^2, \sigma_y = 1.1 \times 10^{-9}\,\text{km/s}^2, \sigma_z = 4.14 \times 10^{-9}\,\text{km/s}^2, k_h = 1.6407 \times 10^{-15}\,1/(\text{km}^2\,\text{s}), k_e = 6.8936 \times 10^{-6}\,\text{km}^2/\text{s}^3$.

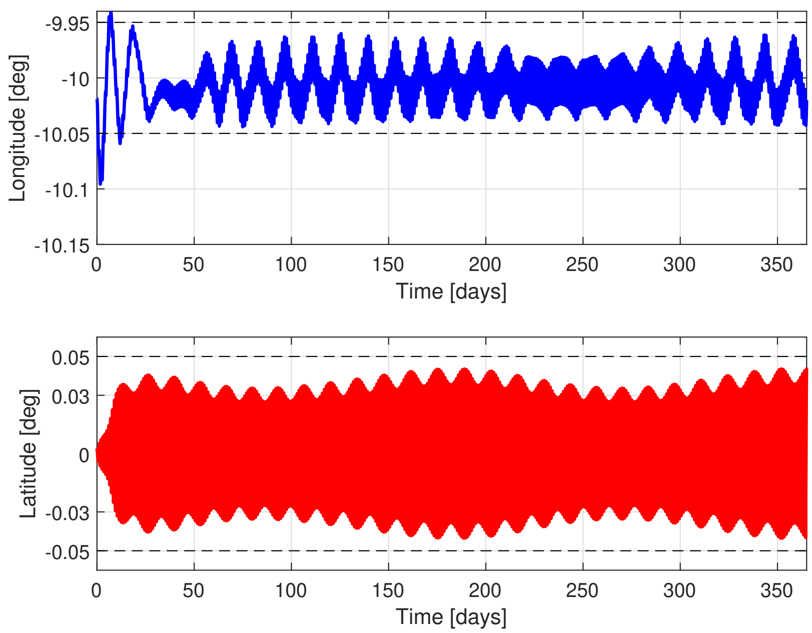

Figure 3: Ground track for optimal discrete thrust

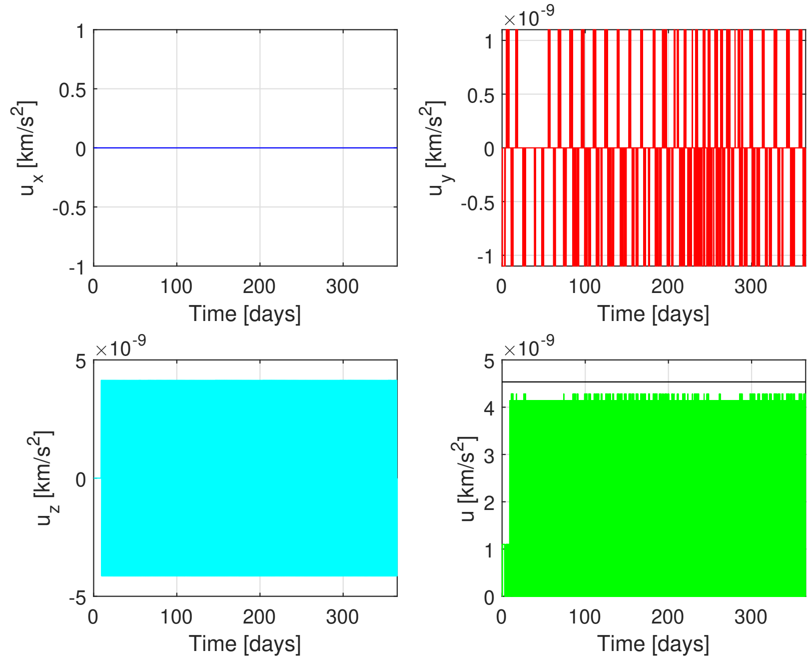

Figure 4: Thrust acceleration components and total acceleration for discrete thrust

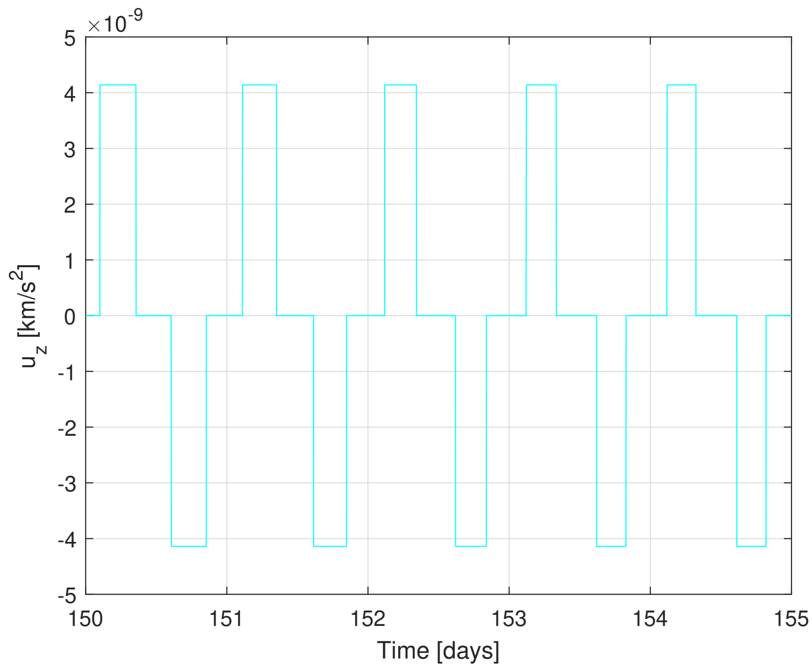

Figure 5: Magnified view of z-axis discrete thrust acceleration

3.3 Optimal Constant Magnitude Thrust

For the optimization of the control law using Constant Magnitude thrust our optimization problem will be:

$$\min_{k_h,k_e}(\Delta V) = \min \int_0^{t_f} \|\mathbf{u}_M\| dt \tag{15}$$

$$s.t. \begin{cases} \delta^2 - \Delta_s^2 & \leq 0 \\ (\lambda - \lambda_d)^2 - \Delta_s^2 & \leq 0 \end{cases} \tag{16}$$

In this research, the other control gains kept as in Ref. [1], namely $k_\lambda = 3200\text{km}^2/\text{rad}, \lambda_b = -0.019°$, and the initial conditions for the simulation were a deviation of 2 km in each ECEF axis from the desired station, and the correct velocity of the desired orbit.

Using MATLAB's `fmincon`function, we were able to search for the ideal set of control gains $[k_h, k_e]$ and found that for $k_h = 2.7147 \times 10^{-15}\, 1/(\text{km}^2\,\text{s}), k_e = 1.043 \times 10^{-5}$ ke the cost is $\Delta V = 56.0879$ m/s/year, thus saving 2.7121 m/s/year compared to Ref. [1].

While maintaining the satellite with the safety area in space, as shown in Fig. 6, with $\Delta\lambda = \pm 0.0437°, \Delta\delta = \pm 0.0288°$, which is less compact on the latitude and the longitude than in Ref. [1] results for the same case.

The control acceleration use is shown in Fig. 7, where the duty cycle for the acceleration on each axis is 46.7%, similar to the duty cycle of z-axis for the discrete thrust method.

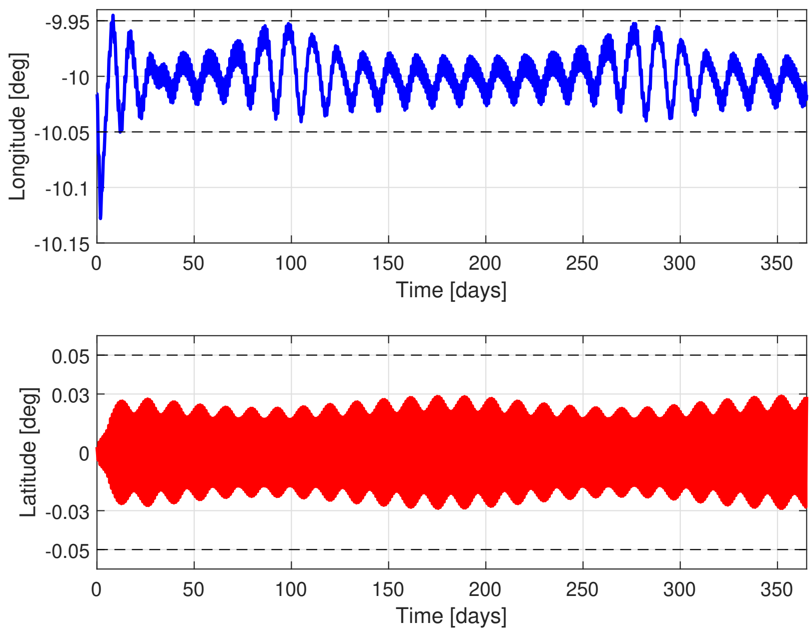

Figure 6: Ground Track For Optimal Constant Magnitude Thrust

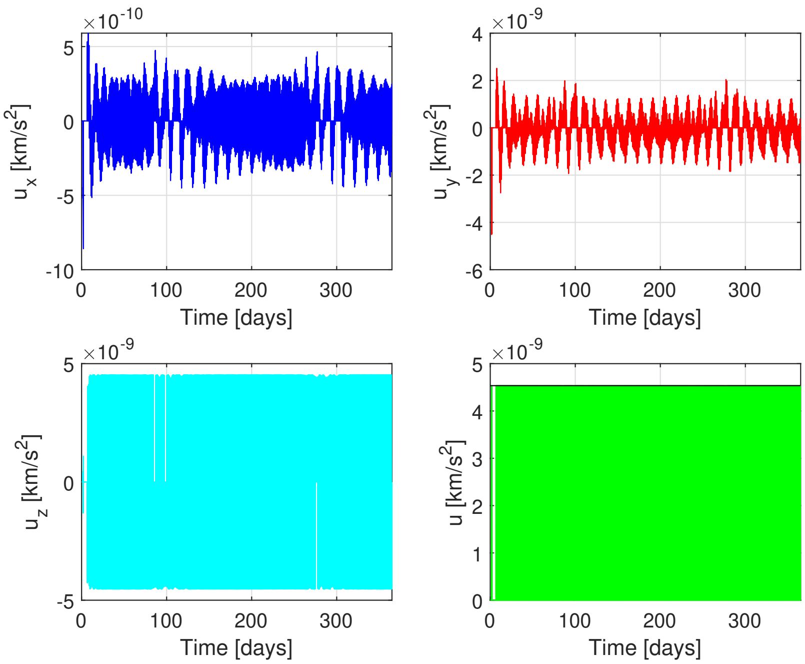

Figure 7: Thrust Components And Total For Constant Magnitude Thrust

4 Result Summary

Using optimization, we were able to find control gains that produced better results in terms of the yearly ΔV than the results shown in Ref. [1], while still keeping the satellite within its original slot in space, although with larger latitude and longitude oscillations.

The optimization worked for three different kinds of thrust implementations – continuous thrust, discrete thrust and constant-magnitude thrust.

The best results were found for the discrete thrust method, as can be seen in Table 1, which managed to keep the satellite within its geostationary slot by using only $\Delta V = 55.55$ m/s/year, thus saving another 7.7 m/s/year compared to Ref. [1], with no use of thrust acceleration in the x-axis and a "bang-bang"-type acceleration in the z-axis with a duty cycle of 46.6%.

The method that was the least optimal is the continuous thrust, which used $\Delta V = 62.795$ m/s/year in order to keep the satellite within its GEO.

Table 1: Results Summary

Thrust Method	**ΔV** [m/s/year]	$\mathbf{k_h}$ [1/(km^2 s)]	$\mathbf{k_e}$ [km^2/s^3]	**Lon x Lat** (°)	**Improvement** [m/s/year]
Continuous	62.795	9.39×10^{-16}	5.48×10^{-6}	$\pm 0.008 \times \pm 0.047$	6.8
Discrete	55.55	1.64×10^{-15}	6.89×10^{-6}	$\pm 0.018 \times \pm 0.044$	7.7
Constant Magnitude	56.088	2.71×10^{-15}	1.04×10^{-5}	$\pm 0.044 \times \pm 0.029$	2.71

5 Future Research

- During the simulation to find the optimal gains, there was a high sensitivity to the initial guess; therefore, future research will focus on Genetic Algorithm in an attempt to find even more accurate optimal gains.

- This paper focused on the long-timescale gains; future research will try to find an optimal short-timescale gains as well.

- Find the optimal combination for control gains and discrete thrust thresholds, while ensuring the stability of the system.
- Modify the control law for a system of collocated satellites.

References

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